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14. ABSTRACT
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Report Title

How Do Cracks Initiate and Grow in a Thin Glass Plate? A Peridynamic Analysis

ABSTRACT

In this report, we present peridynamic simulation results and experimental evidence for the origin and evolution of cracks in a thin glass plate that is impacted near its center, at moderate speeds (150m/s), by a small steel projectile. A polycarbonate thin backing plate is used to preserve location of glass fragments. In the computational model the tape used on the sides to hold the two plates together is absent. Upon impact, a series of circular cracks form on the strike face, followed by radial cracks advancing from the impact site towards the boundaries, driven by elastic deformations induced behind the Rayleigh wave. At the same time, a through-thickness Hertz-crack grows, as well as cracks parallel to the sides of the plates, induced by waves reflected from the boundaries reinforcing surface waves emanated from the impact region. We present evidence gathered from our simulations for the origin and evolution of these cracks, and confirm these results with fractography experiments of post-mortem samples. The results provide evidence of the predictive capabilities of the peridynamic model for simulating complex dynamic fracture behavior in brittle targets and offer arguments for what the sufficient ingredients in a model for dynamic brittle fracture should be.

Conference Name: U.S. National Congress on Theoretical and Applied Mechanics 2014

Conference Date: June 17, 2014

How Do Cracks Initiate and Grow in a Thin Glass Plate? A Peridynamic Analysis

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- ARL (Dr. C.F. Yen)
- ARO (Dr. Ralph Anthenien)
- AFOSR MURI

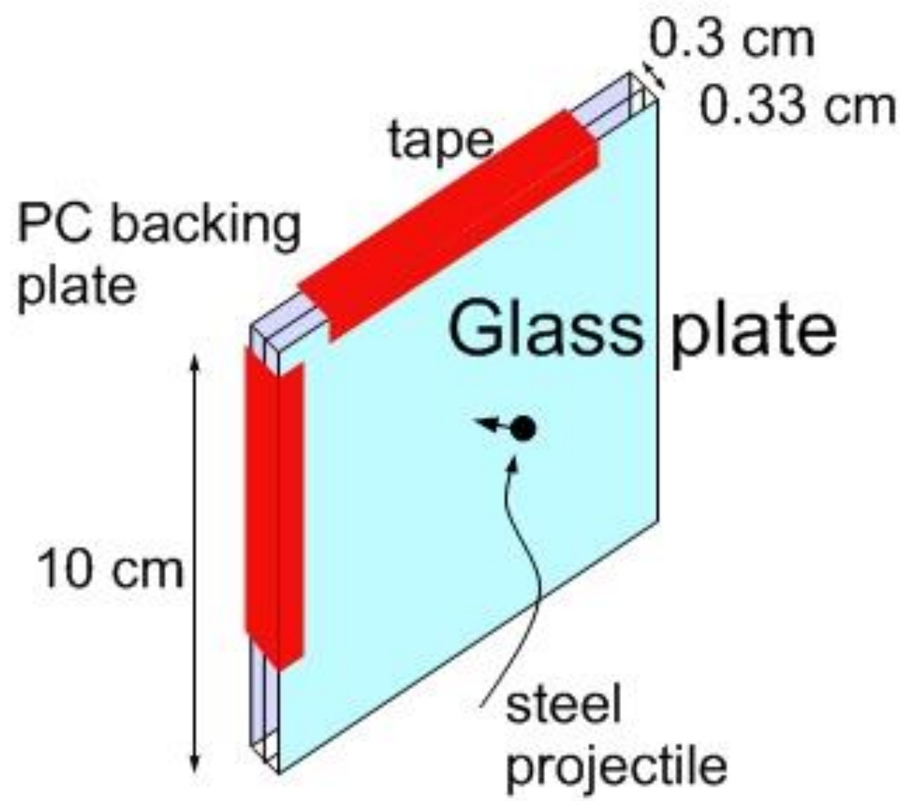
(Drs. David Stargel, Fariba Fahroo)

Experimental Setup

- Plates suspended to mimic "no boundary conditions".
- Impact speeds of up to 150m/s. No damage observed in PC plate.
- Tape is used along boundaries to recover fragments after impact.
- Impact location is off-center

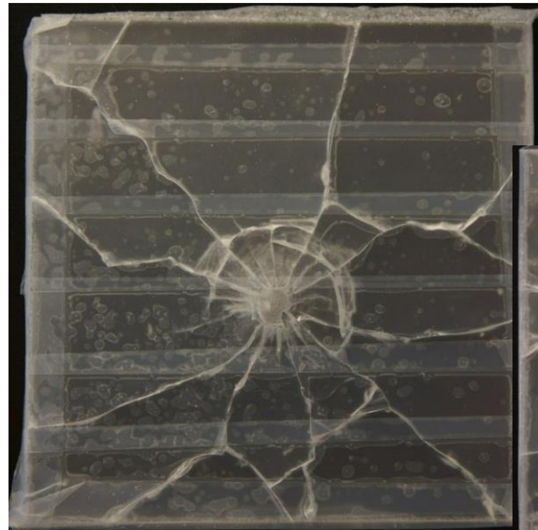
(1 cm closer to the right).

Experiments by
J. Yu, C.F. Yen (ARL)



Experimental Results

- Pictures of strike face (taken by Y. Wang at UNL)
- Tape is applied on the impact face only post-mortem, to preserve fragments.
- At higher impact speeds, fragments are dislodged and slide out of place being caught between the two plates.



Impact speed 58 m/s

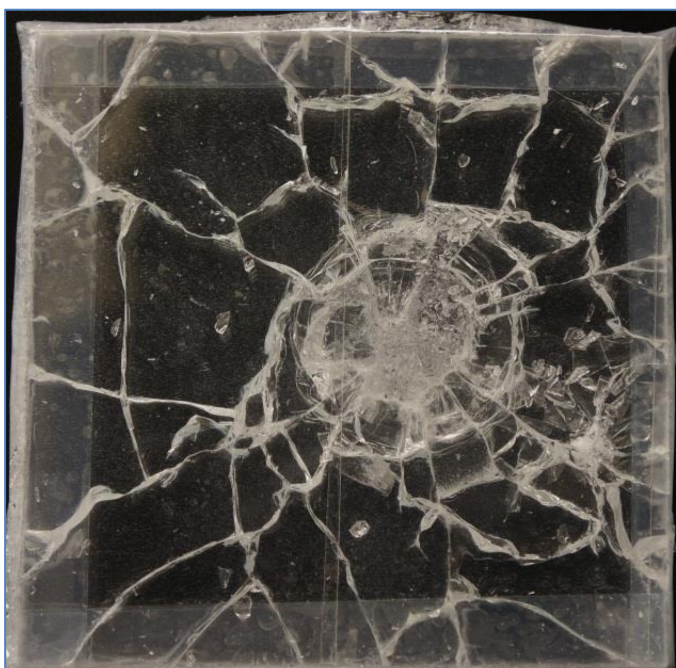
Impact speed 100 m/s

Impact speed 150 m/s

Hu, Wang, Yu, Yen, and Bobaru, International Journal of Impact Engineering (2013)

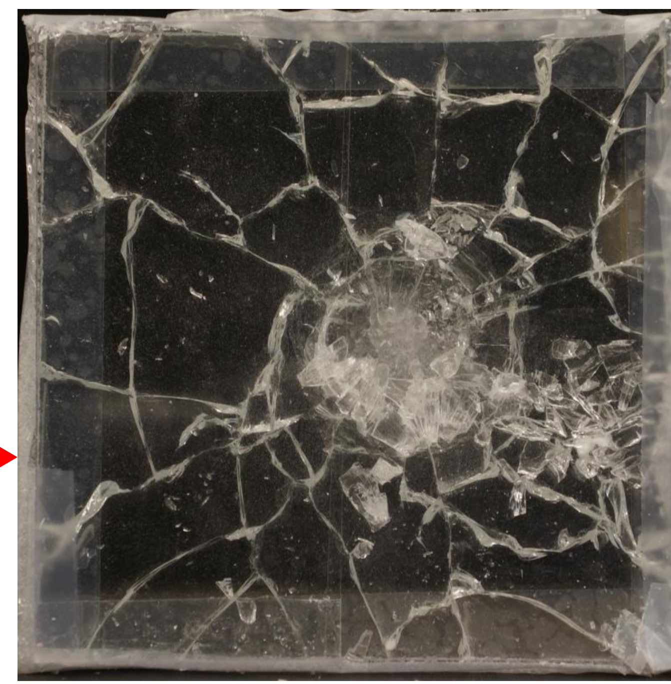
Strike and back faces of glass at 150m/s

- Between 33-35 major fragments at 150m/s impact speed; many through-thickness cracks are tilted.
- Major radial cracks, some “branch” before reaching the boundaries.
- Impact cone: small region of comminuted material on strike face, more damage on the back face of the plate.
- Major circumferential cracks, and some surprising, very fine, wispy lines/cracks up to 3.8-4 cm diameter around the impact center.
- Some through-thickness cracks are parallel to the sample boundaries.



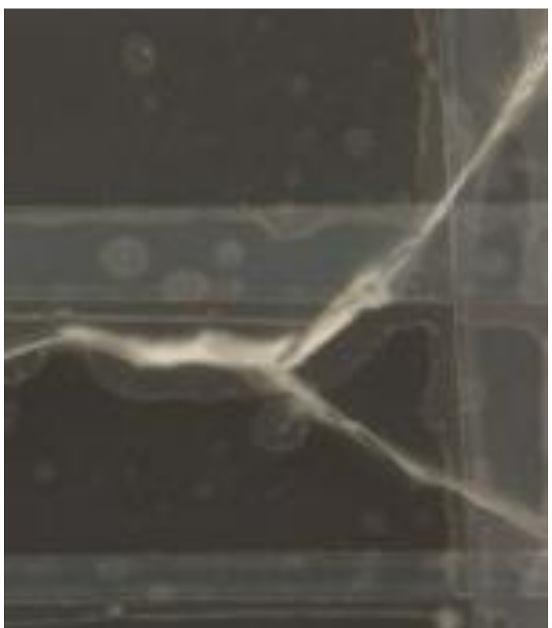
Strike face

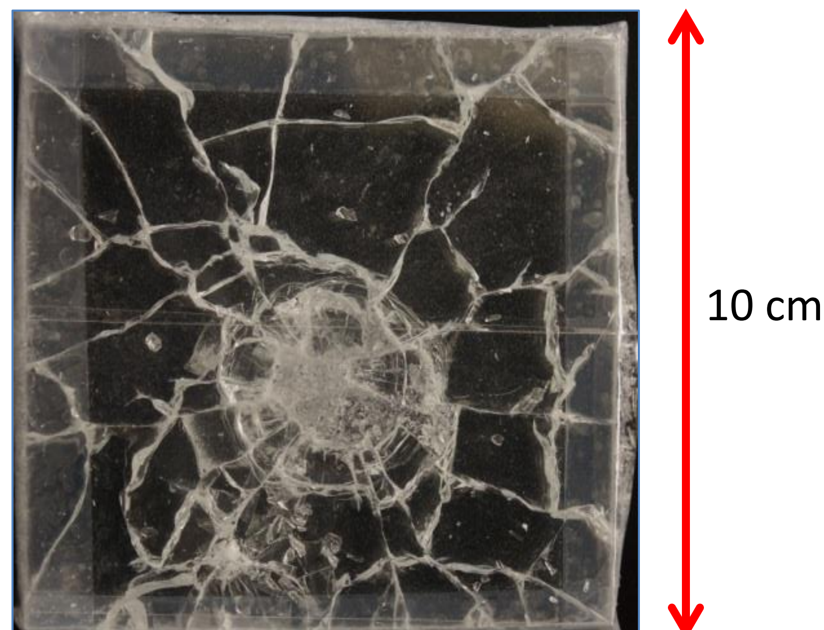
Back face



Questions to be answered

- Can we understand how and why each type of crack system forms?
  - Crack surface fractography can give indication of crack speed, crack growth direction, order or crack generation
- Can we compute such complex behavior?
- “Complexity-free fracture/fragmentation/damage”. Can a single model, with minimal inputs, solve the problem of one crack and that of many cracks from impact?





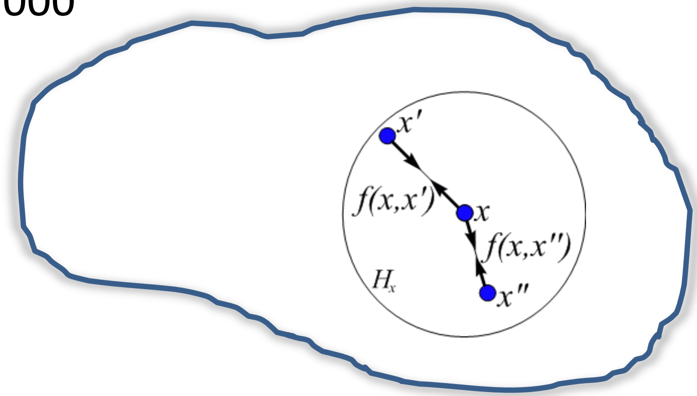
The Peridynamics formulation

S.A. Silling *JMPS* 2000

Replace the stress-divergence term in

$$\rho \ddot{\mathbf{u}} = div\ \sigma + \mathbf{b}$$

by an integral of forces

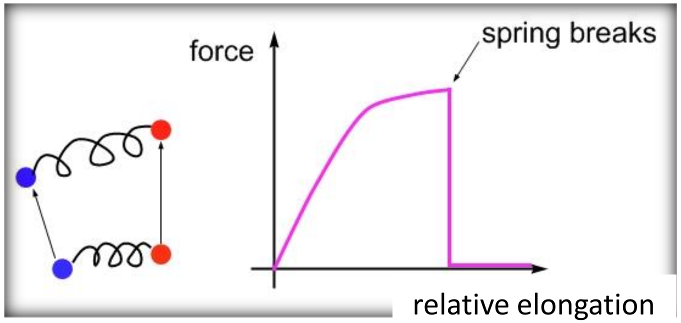


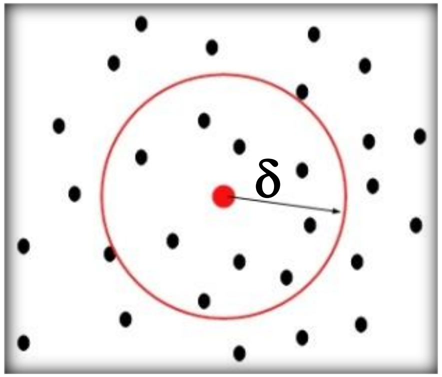
$$\rho \ddot{\mathbf{u}} = \int_{H_x} \mathrm{f}\,(\mathbf{u}(\mathbf{x}', t) - \mathbf{u}(\mathbf{x}, t), \mathbf{x}'\text{-}\mathbf{x})\ dV_{\mathbf{x}'} + \mathbf{b}$$

$\mathbf{f}(\eta, \xi)$ is a pairwise force (force density on particle x due to particle x')

$\xi = x' - x$ (original relative position)

$\eta = (u' + x') - (u + x) = \Delta u + \xi$ (current relative position)





A linear micro-elastic material

$$\mathbf{f}(\eta, \xi) = \frac{\partial w(\eta, \xi)}{\partial \eta}$$

$$w(\eta, \xi) = \frac{c(\|\xi\|)\, s^2 \|\xi\|}{2}, \qquad s = \frac{\|\xi + \eta\| - \|\xi\|}{\|\xi\|}$$

relative elongation

Introduce **damage**

$$\hat{f}(s, |\xi|, x, t) = \bar{f}(s, |\xi|)\ \mu(\xi, x, t)$$

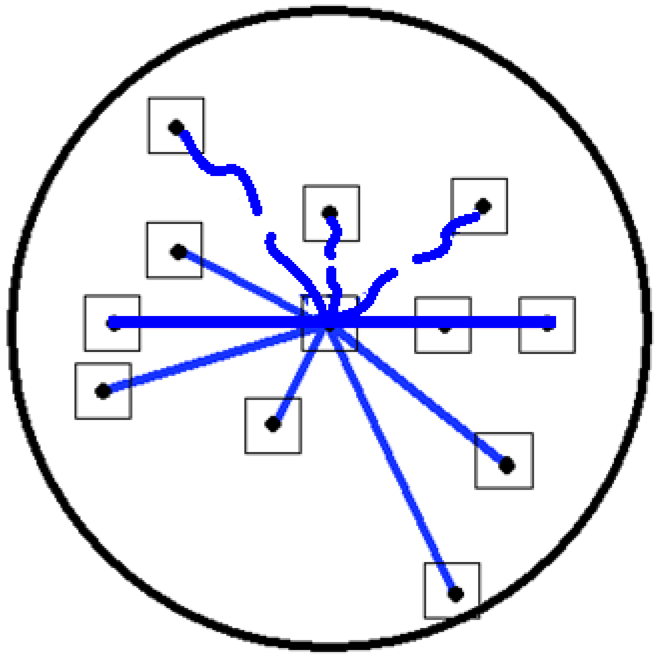
with $\mu(\xi, x, t)$ a history - dependent scalar 0 - 1 function

$$\mu(\xi, x, t) = \begin{cases} 1 & \text{if } s < s_0 \text{ for all } 0 \le \tau \le t \\ 0, & \text{otherwise} \end{cases}$$

(**Silling and Bobaru** *Int. J. Nonlin. Mech.* **2005**)

How cracks form in peridynamics

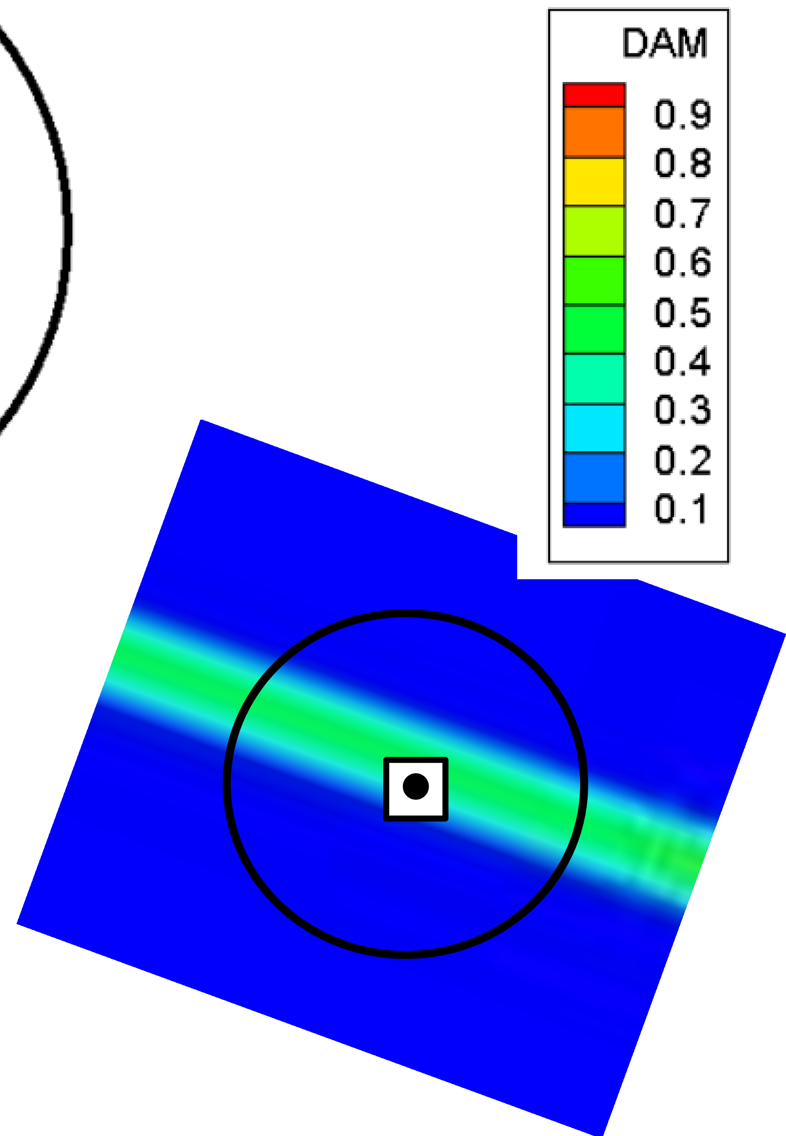
S.A. Silling (2000)
Silling and Askari (2005)



Damage index number

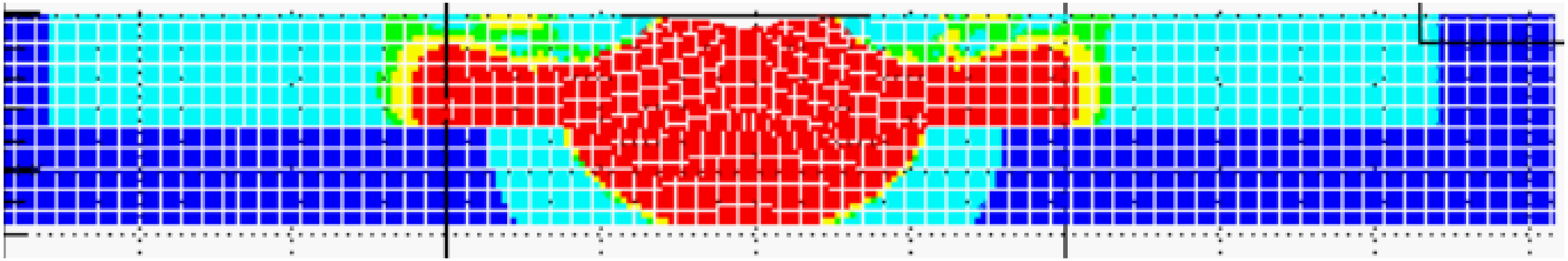
$$d = \frac{n_{\text{broken}}}{n}$$

- d is between 0 and 1.
- Damage index of around 0.4-0.5 localized along a line (in 2D) or surface (in 3D) indicates that a crack has formed.



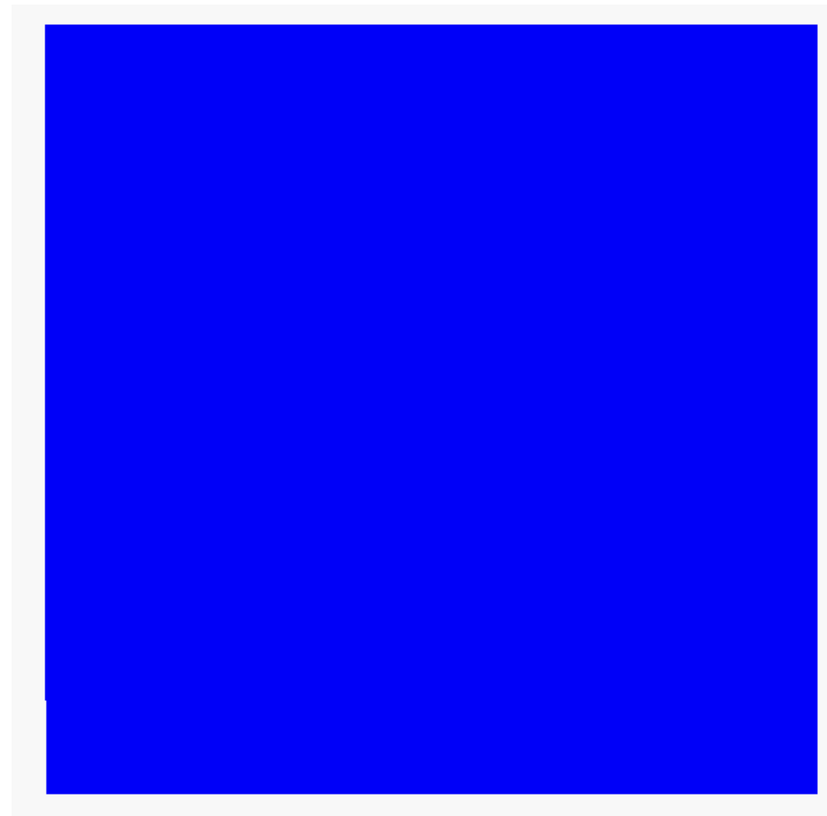
Peridynamic model

- No tape on sides of the plates in computations (PC plate flies away from glass after impact).
- Side-taping: rebound of PC plate causes further damage in exp.
- Use 15 nodes through-thickness of glass; compute 100 μs after impact. Grid spacing 250 μm, nonlocal horizon size ~1mm.
- Notice different wave speeds in glass and PC (below: strain energy density for a portion cross-section through impact point)



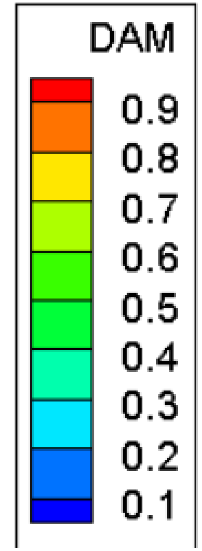
Evolution of damage

Strike face

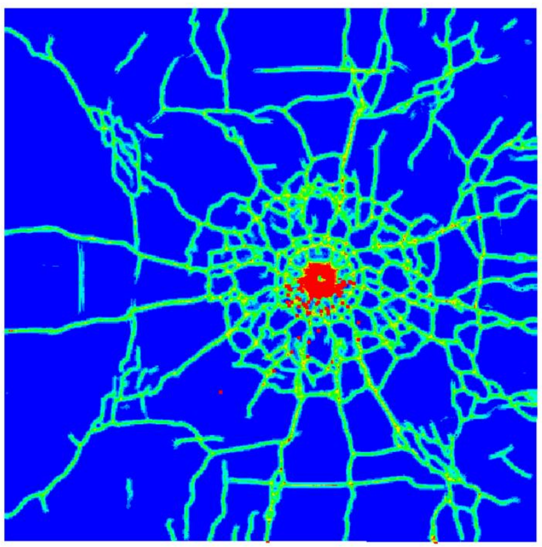


Back face

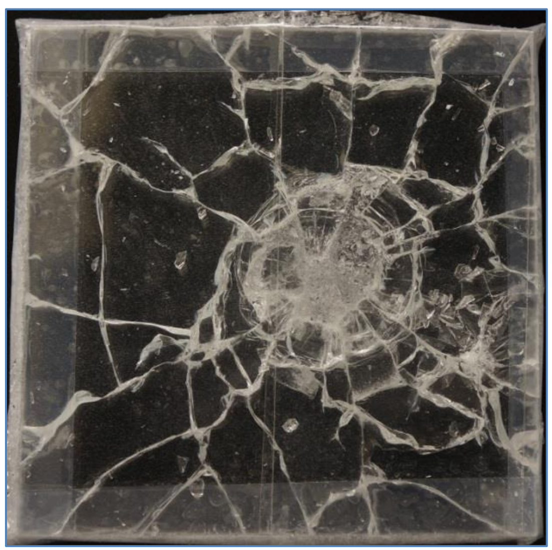
Peridynamic results for impact speed 150m/s: damage at 100 μs after impact

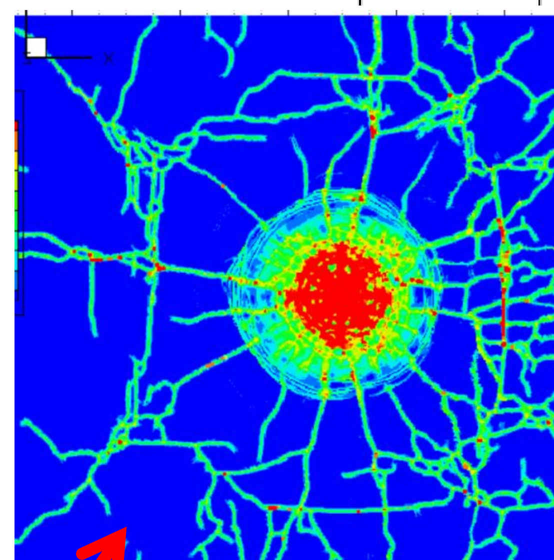


- Similar number of major fragments as in experiments (~33-35)
- Similar structure of cone fracture, radial cracks, and circumferential cracks as in exp.
- Through-thickness cracks parallel to the side boundaries, like observed in experiments.
- Some cracks “branch” as they approach the boundaries.
- A set of wispy “cracks” seen on the back face: same outer diameter as in experiments (~3.8-4 cm), non-symmetrical.



Strike face

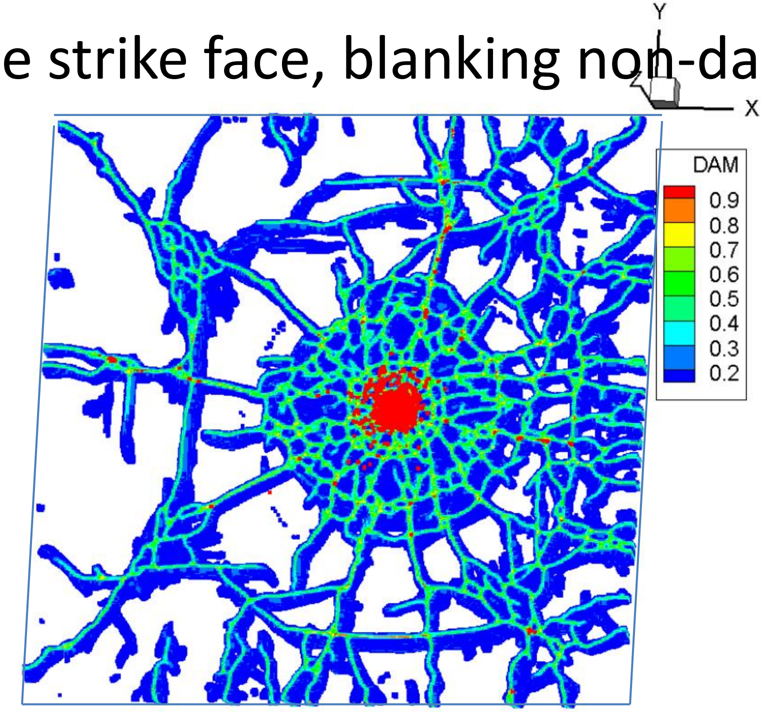




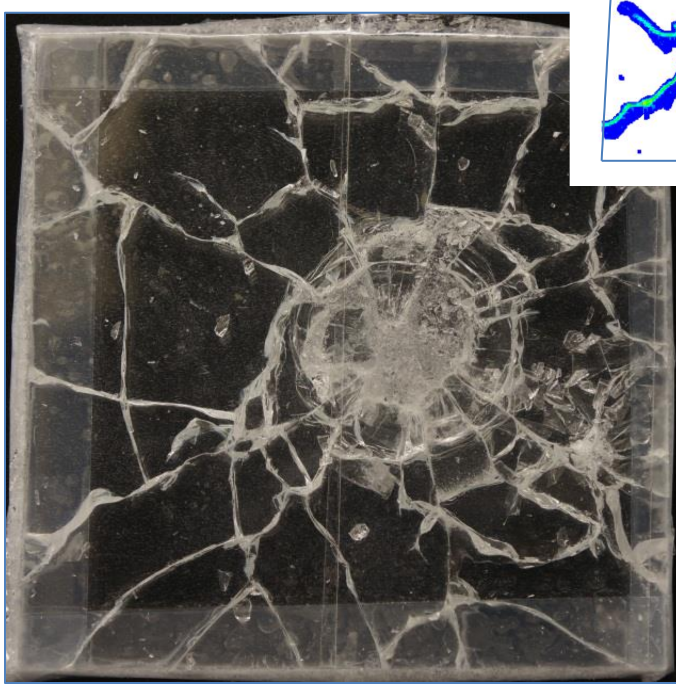
Back face

A 3D view of Peridynamic damage

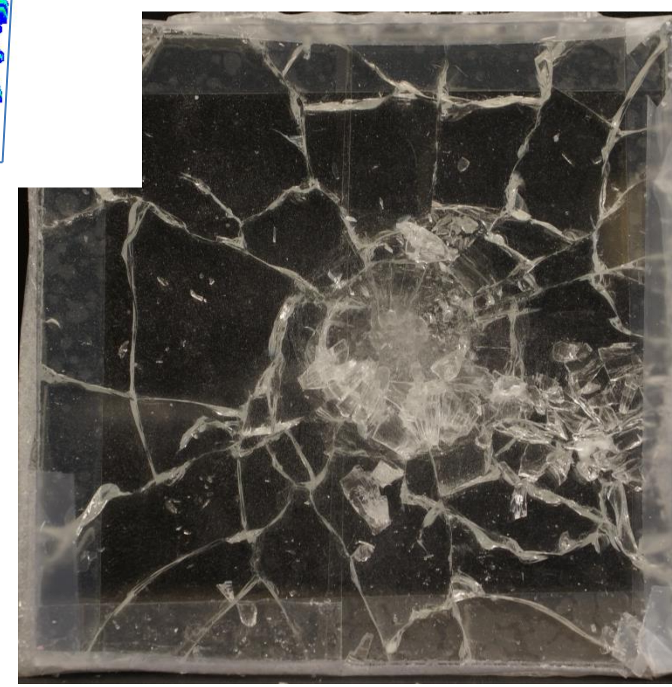
Tilted view from the strike face, blanking non-damaged nodes



Strike face

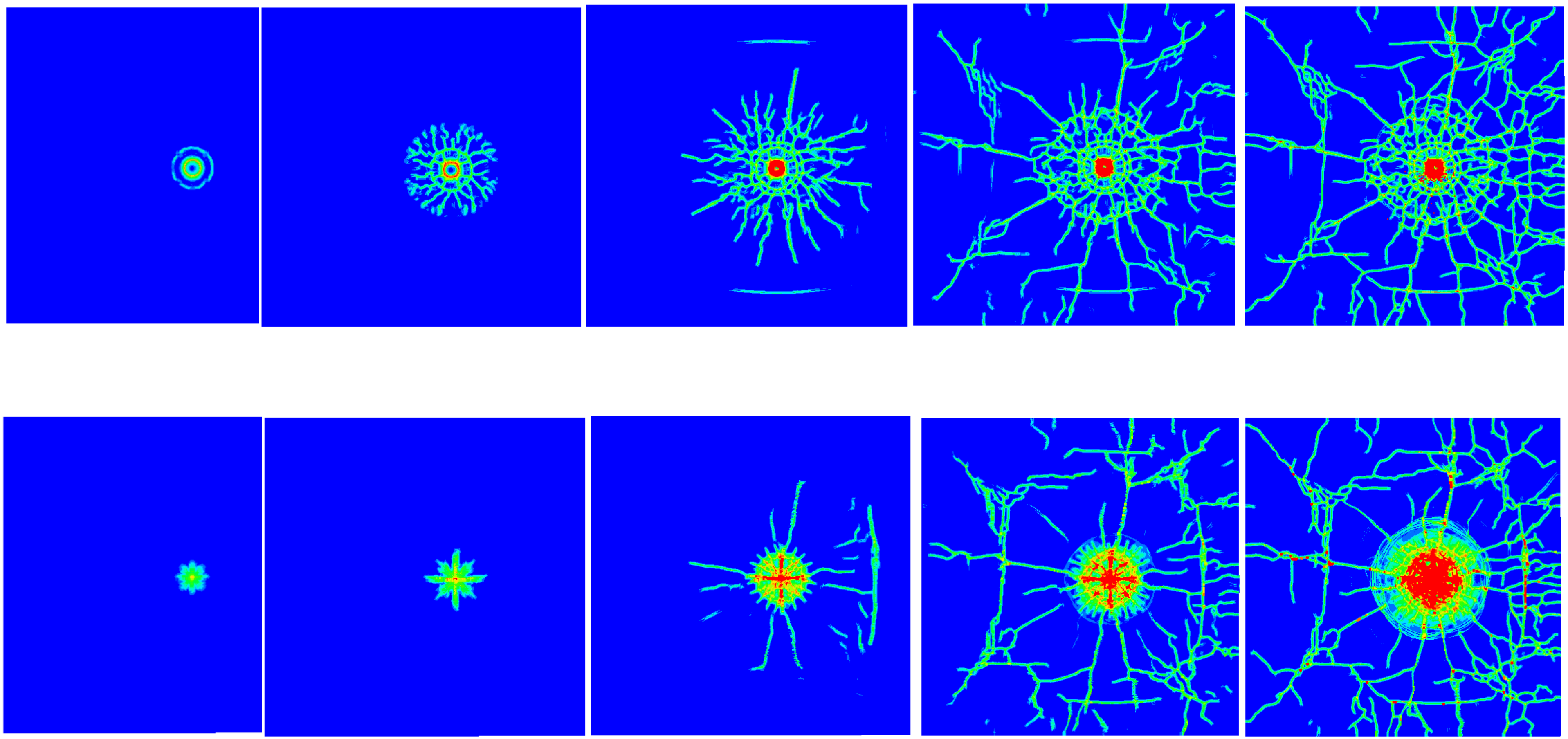


Back face



The evolution of damage

Hu, Wang, Yu, Yen, and Bobaru, International Journal of Impact Engineering (2013)

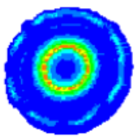


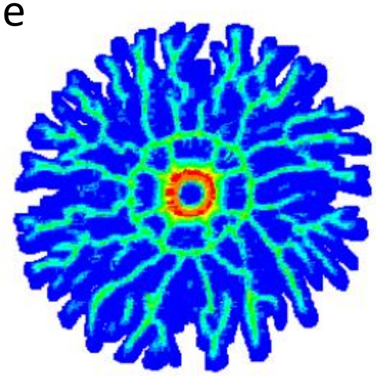
Damage map for top (top row) and bottom (bottom row) glass surfaces at 2.95, 7.25, 15.5, 30.35, and 99 μs from the time of impact.

Evolution of damage

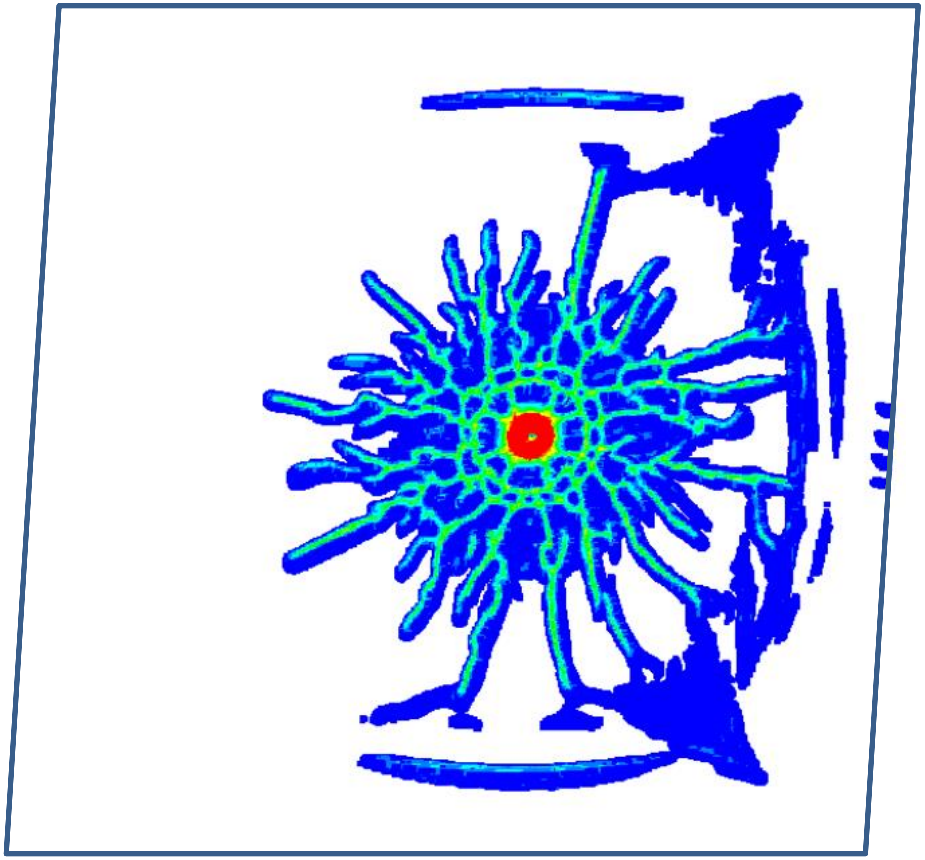
- View from the strike face

Two ring-cracks, match theoretical and experimental results. Starting of Hertzian cone



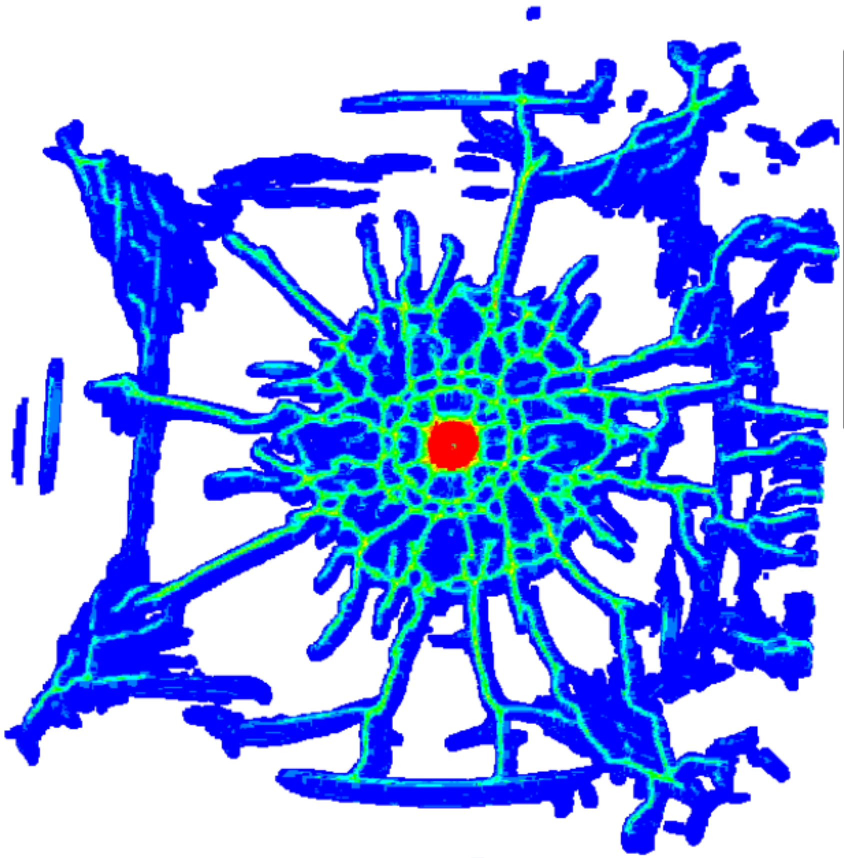


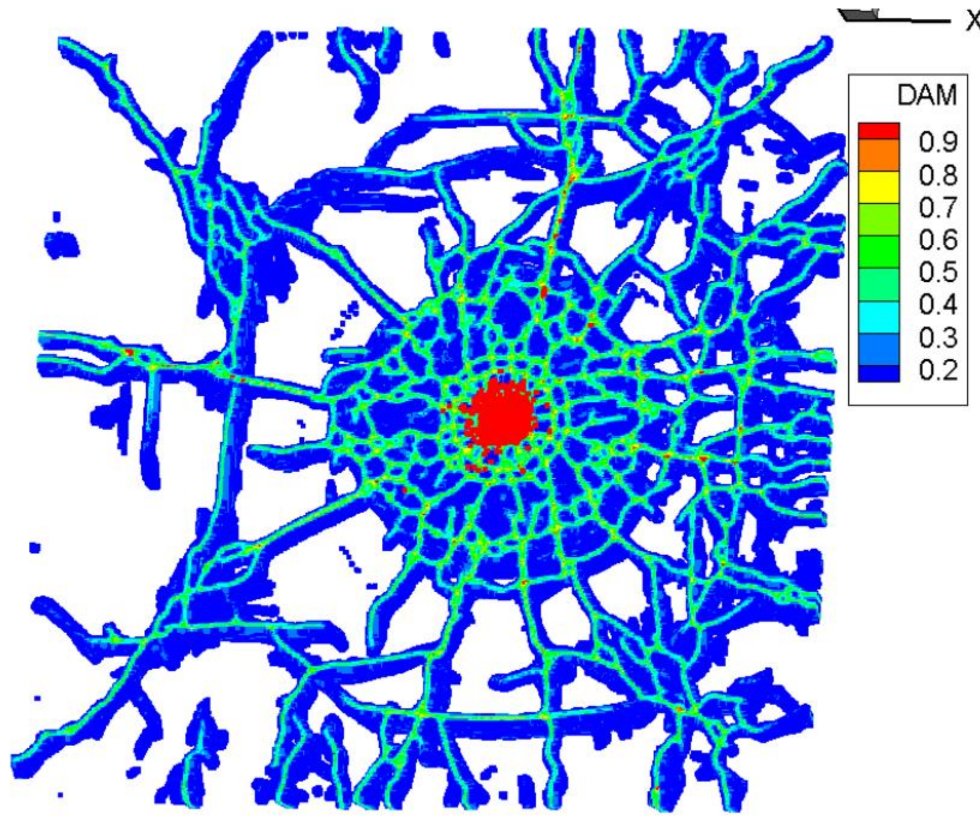
Transition from ring cracks to localized radial fractures, that branch, happens due to hoop stress becoming critical behind the Rayleigh wave.



Reflected waves from the boundaries generate through-thickness cracks parallel to the sides

The evolution of damage (cont'd)

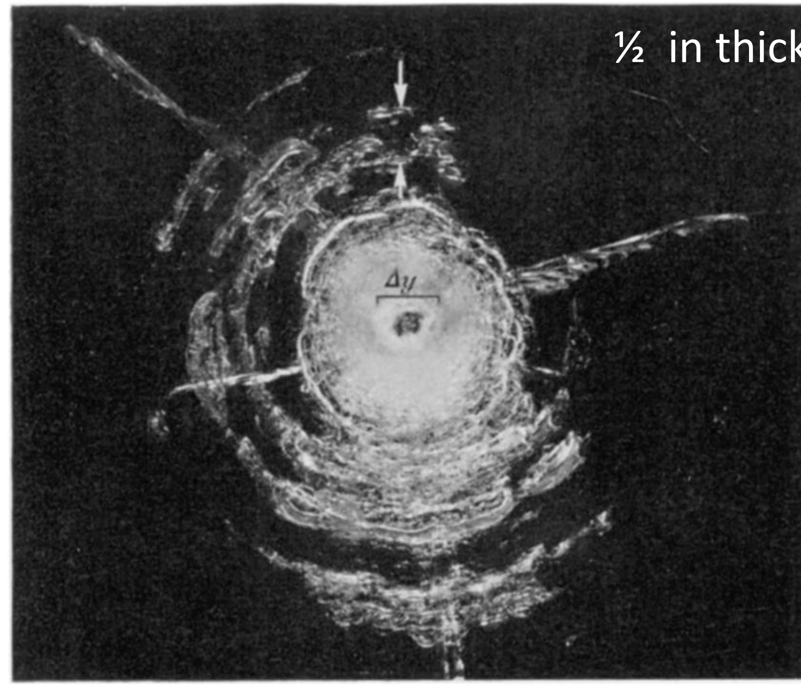


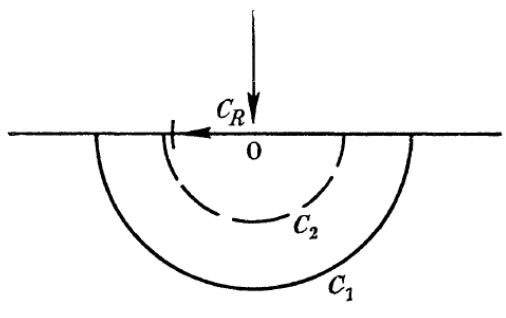


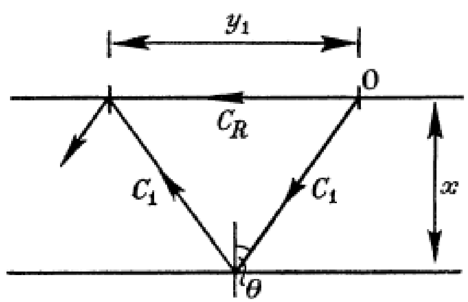
Some cracks start from the boundaries
and join with radial cracks growing from the center

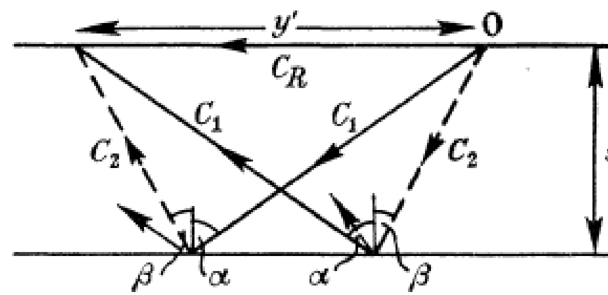
The Hertzian cone transitions to a crack surface parallel to strike face, and wispy roughness forms on this surface.

Ring (band) cracks in thin plates



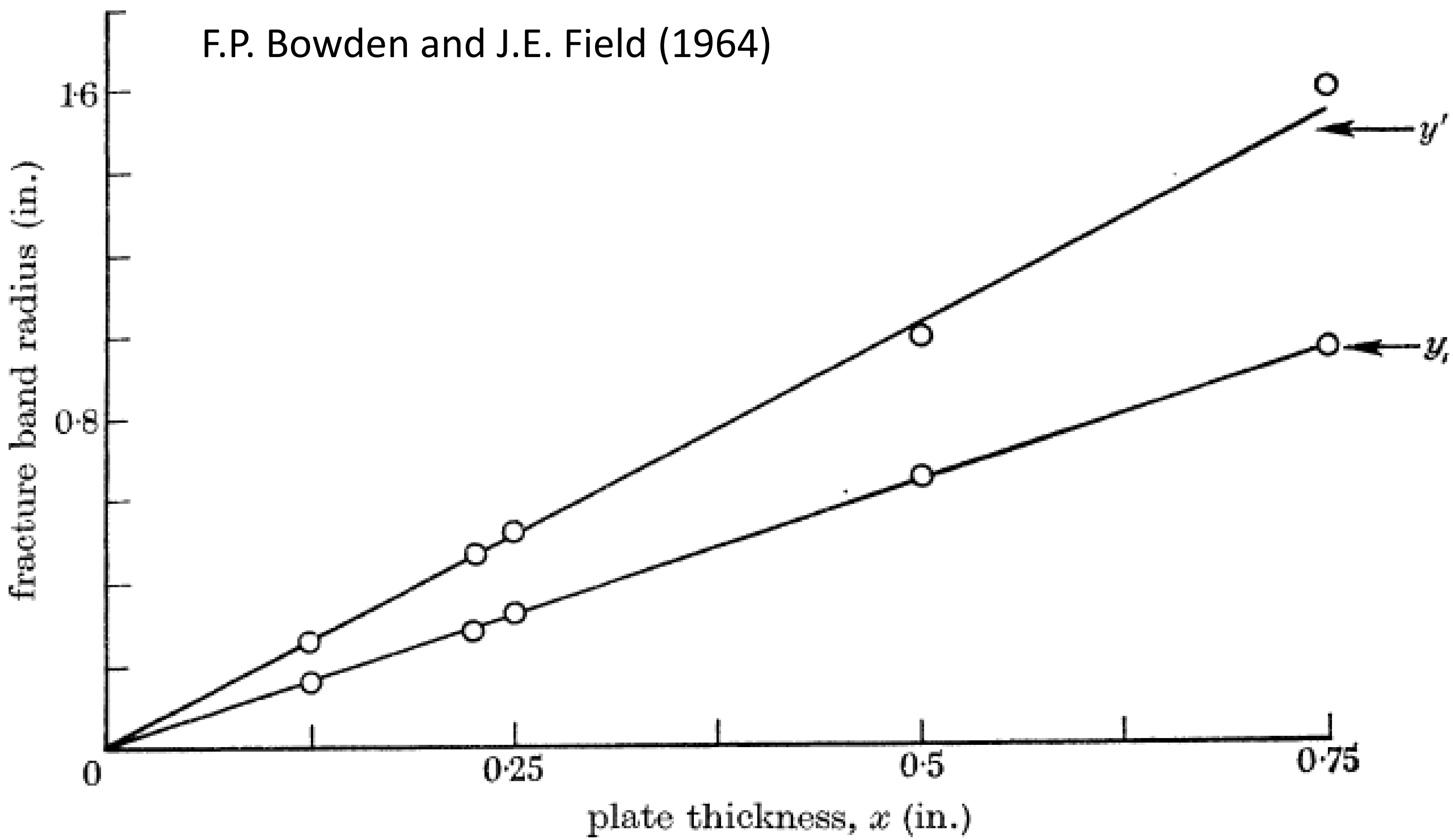






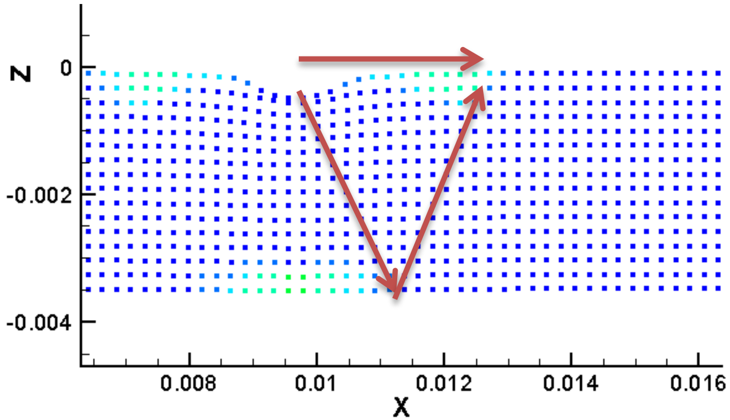
F.P. Bowden and J.E. Field, *"The brittle fracture of solids by liquid impact, by solid impact, and by shock."* Proc. Royal Soc of London, A, **282**: 331-352 (1964)

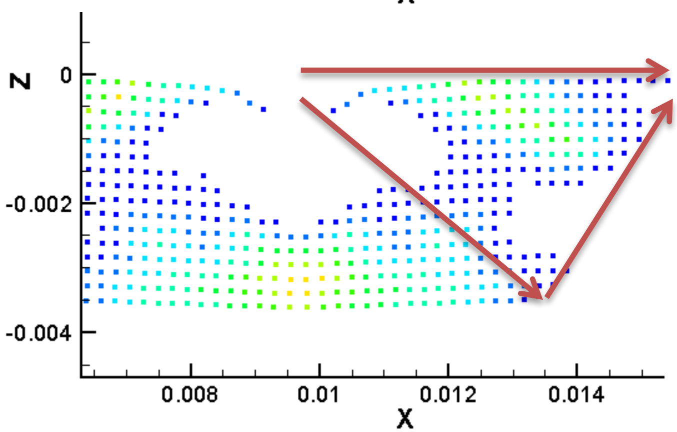
F.P. Bowden and J.E. Field (1964)

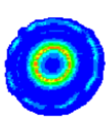


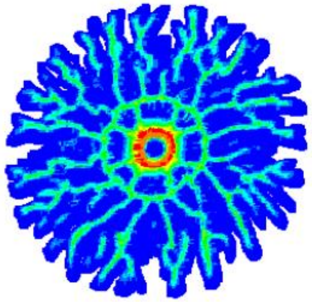
Peridynamic results for ring cracks

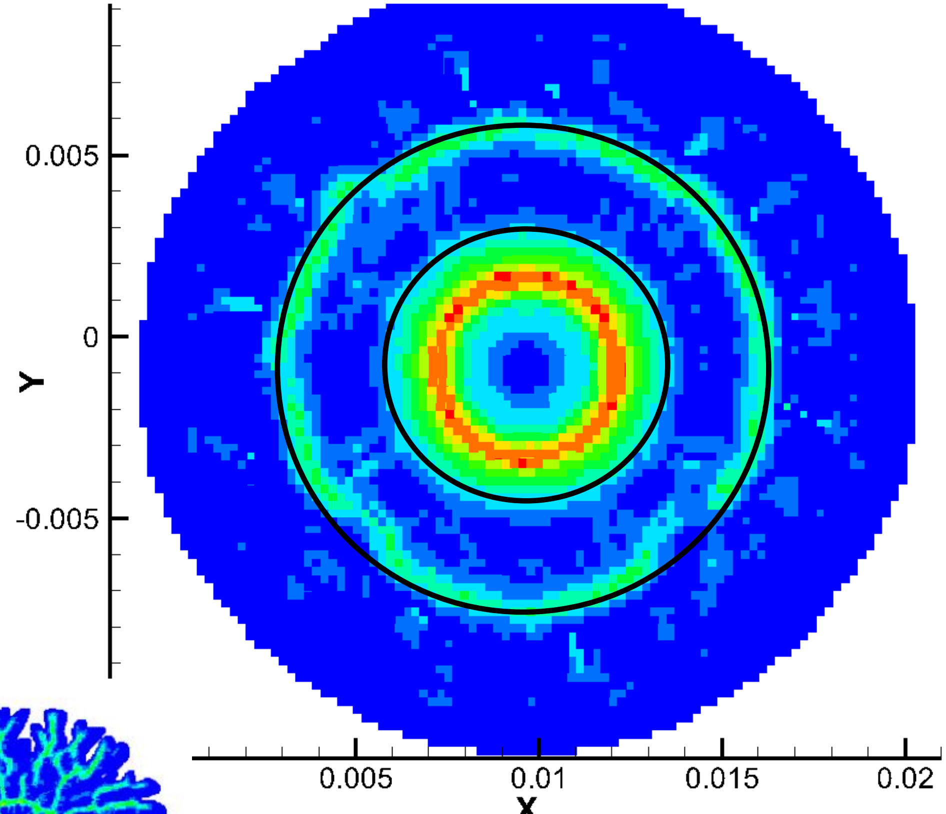
- For a 3.3 mm thick soda-lime glass plate, **theory** predicts first radius of 4.2mm, second radius of 6.6mm (black circles).
- **PD results**: first radius of ~4mm, second radius of ~6.2mm



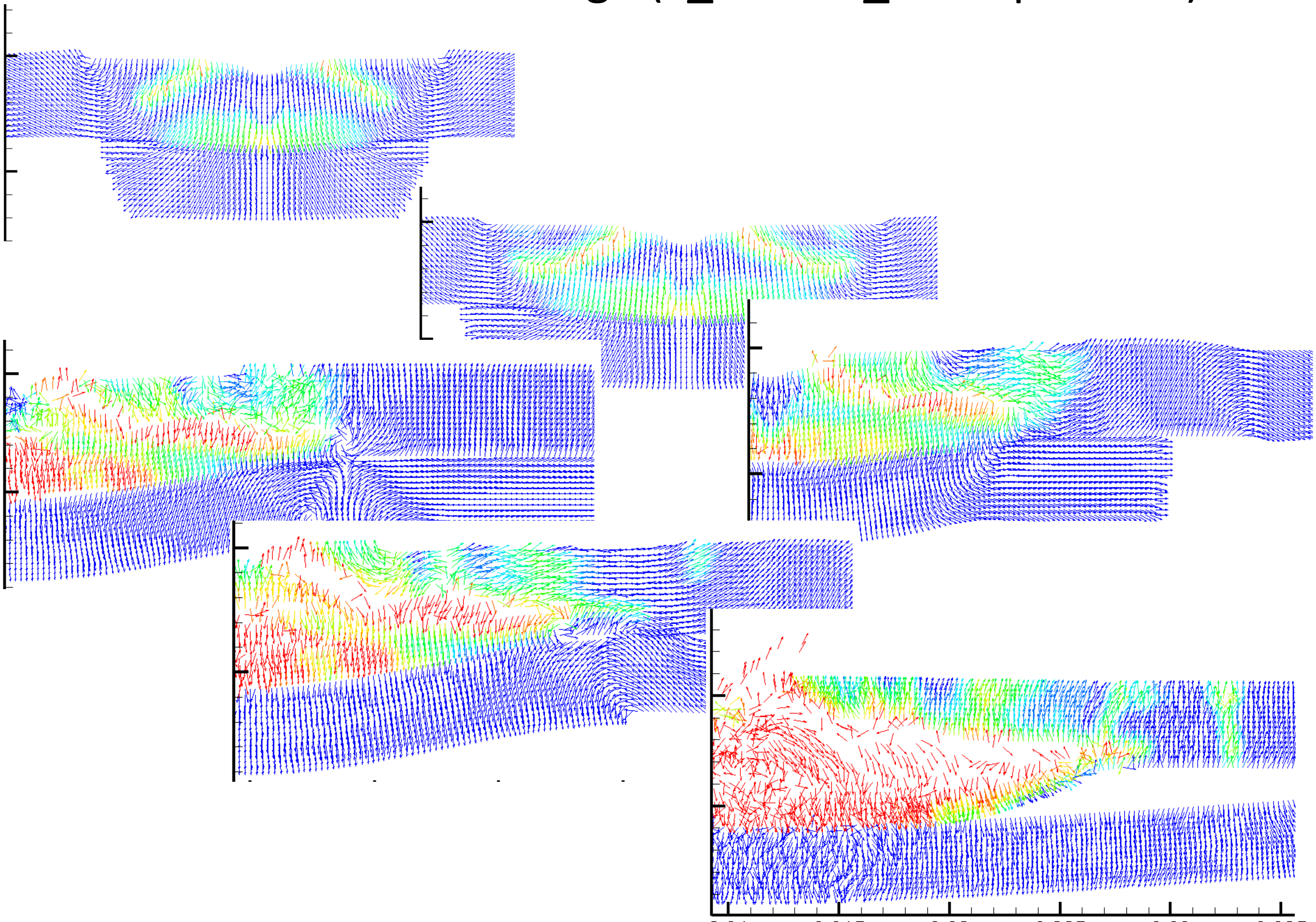






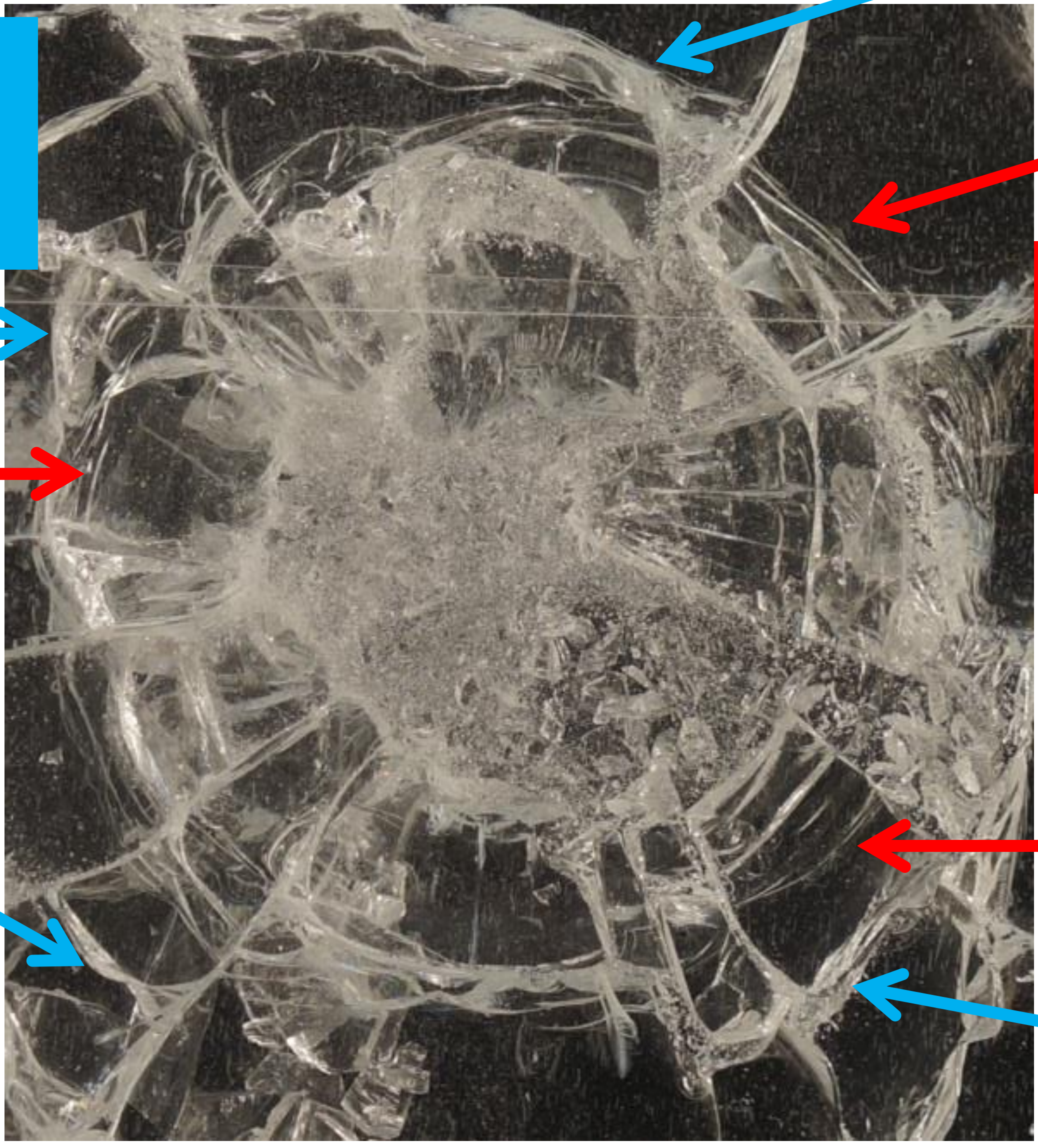


The evolution of damage (v_x and v_z components)



What are the wispy circumferential “cracks” ?

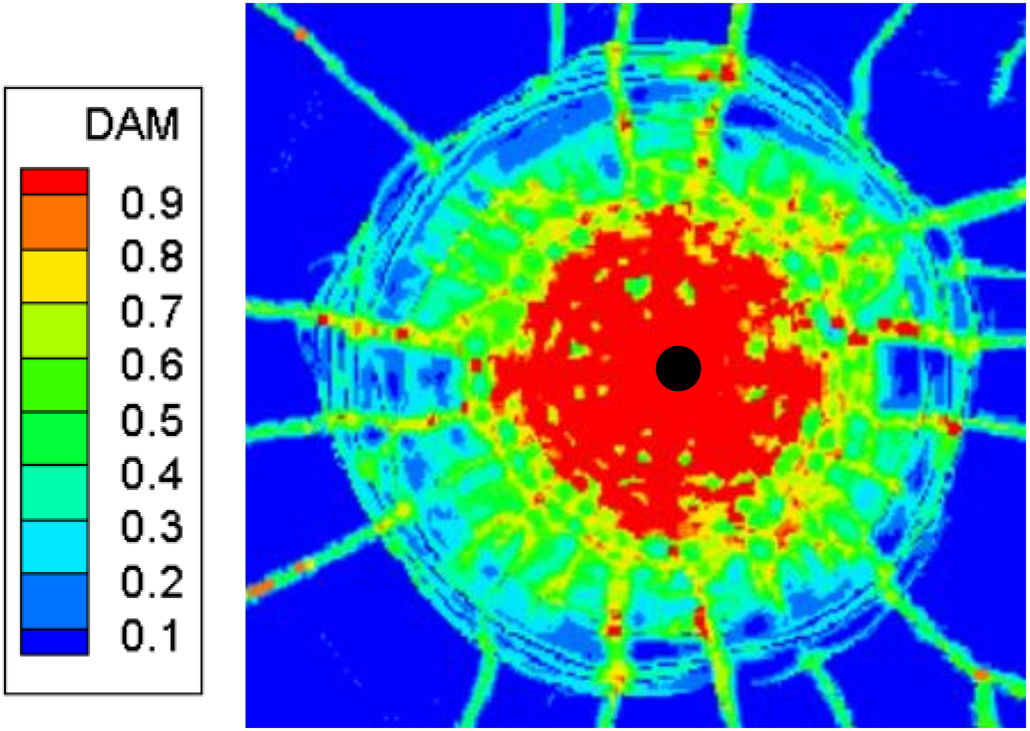
Coarse circumferential cracks

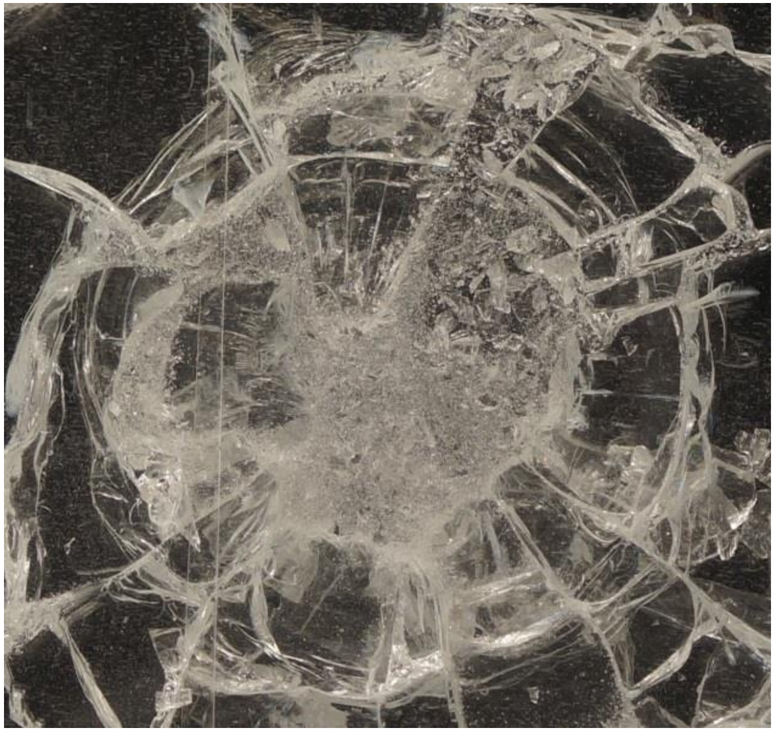


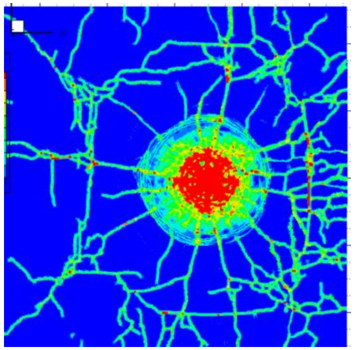
Wispy circumferential “cracks”

Impact speed 150 m/s

The region around the impact point (back face)

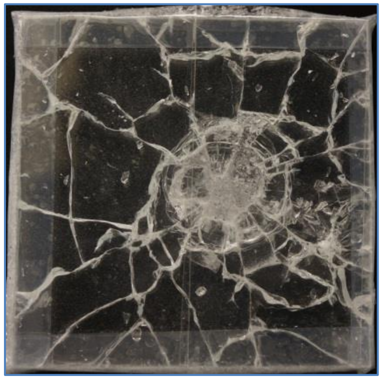






Damage on
back side layer of nodes

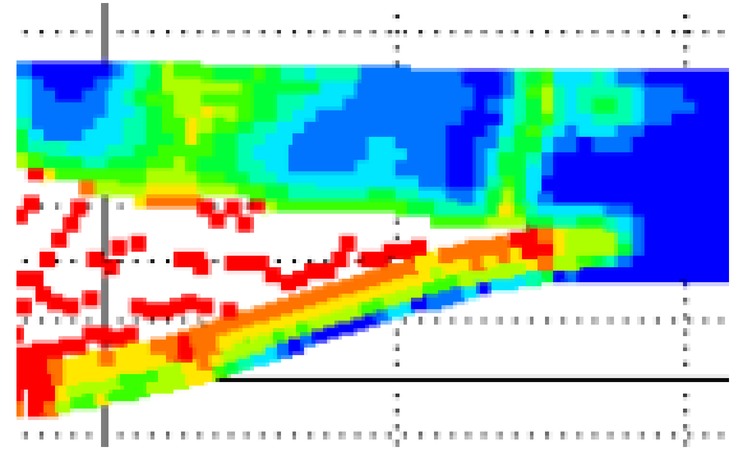
See-through
from front
side

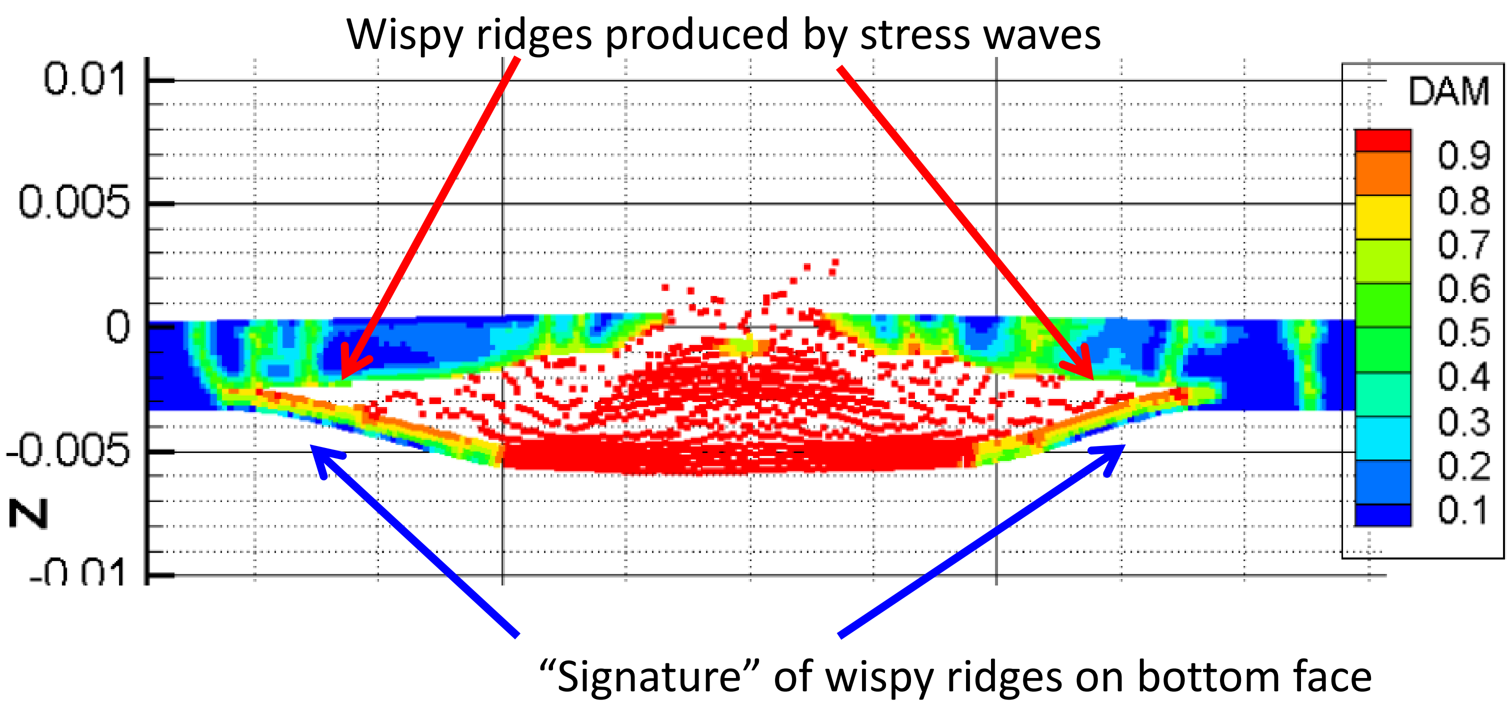


Outer diameter for
wispy "cracks": ~3.8-4 cm

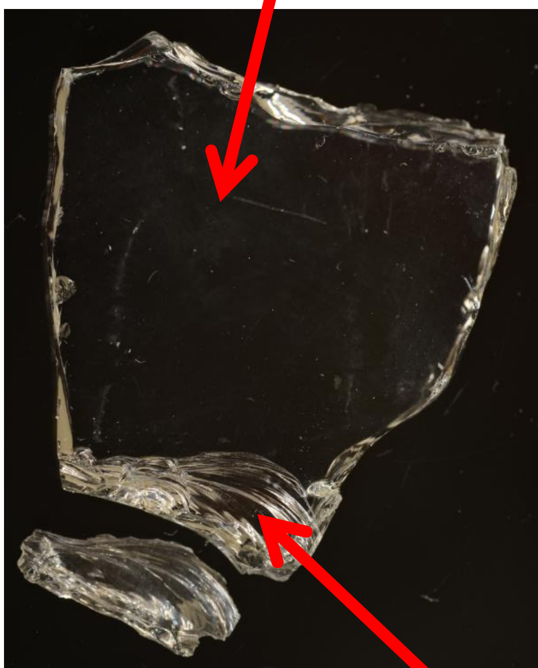
Surprising find !!!

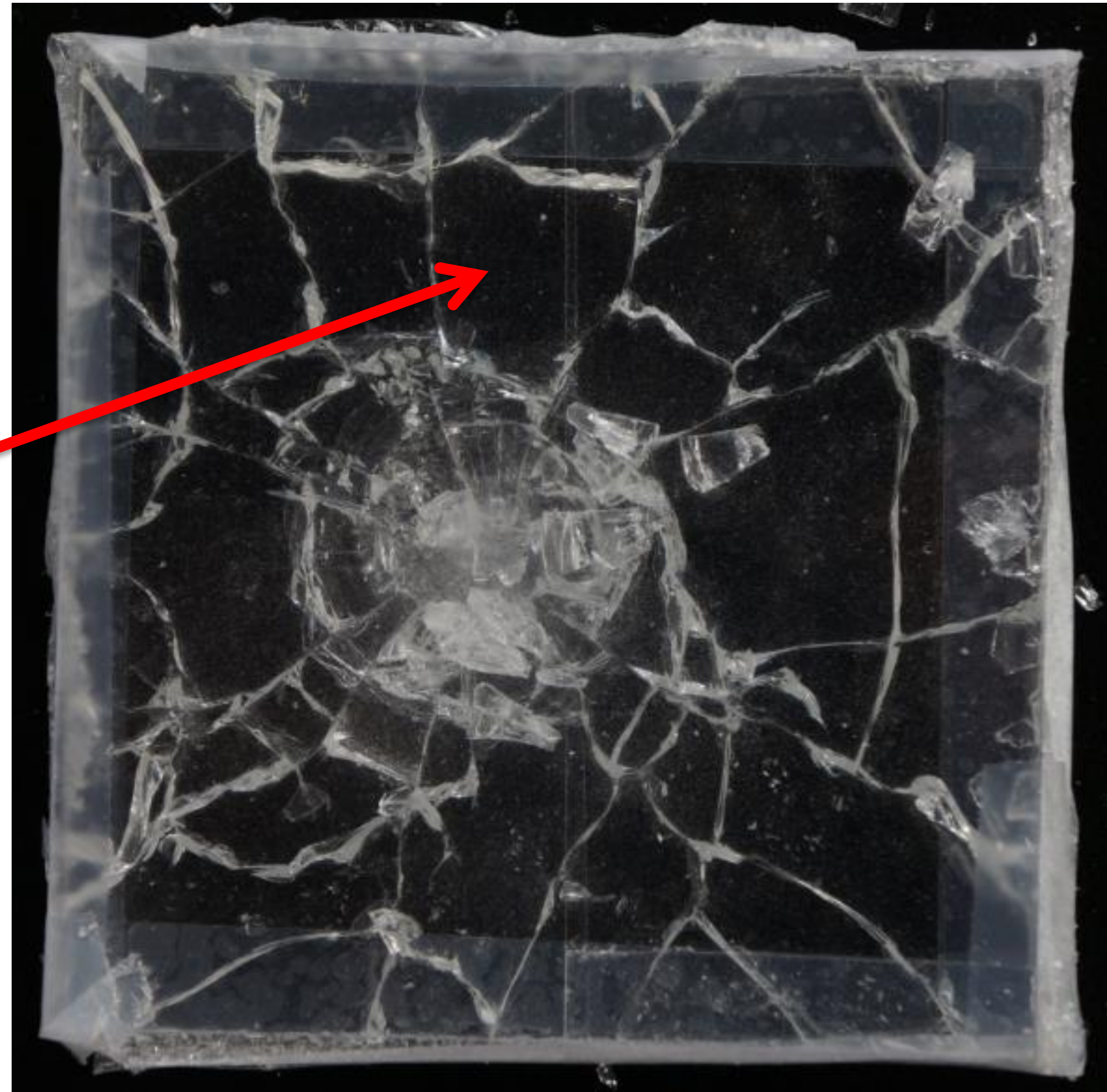
- Computed wispy “cracks” are not through-thickness cracks! Damage index not close to 50% for nodes on back-face.
- Damage seen as wispy “cracks” is signature of roughness on cracks parallel to strike face.





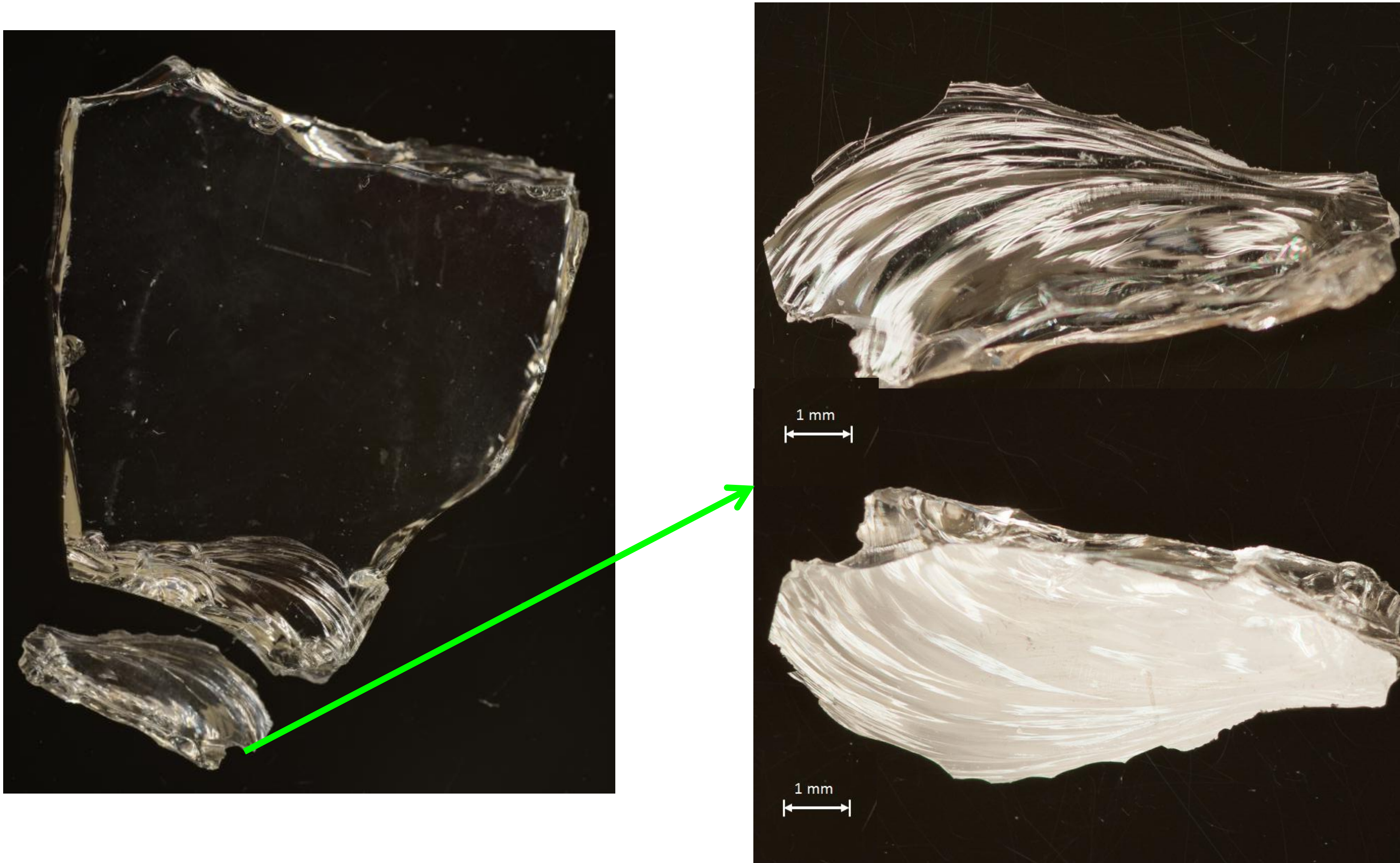
Large fragment and glass chip formed by the transverse crack





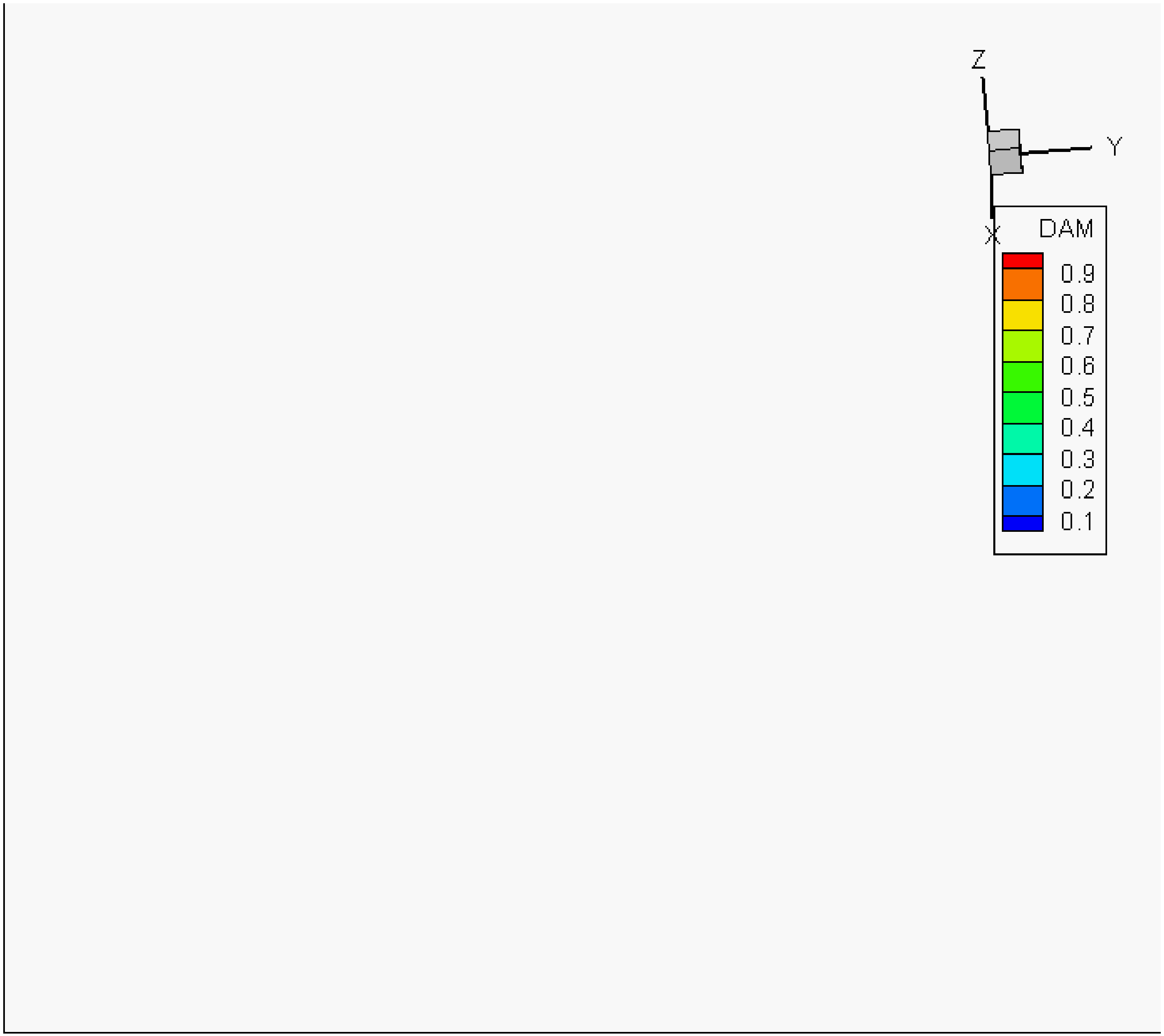
Wispy roughness of the transverse crack that created the chip

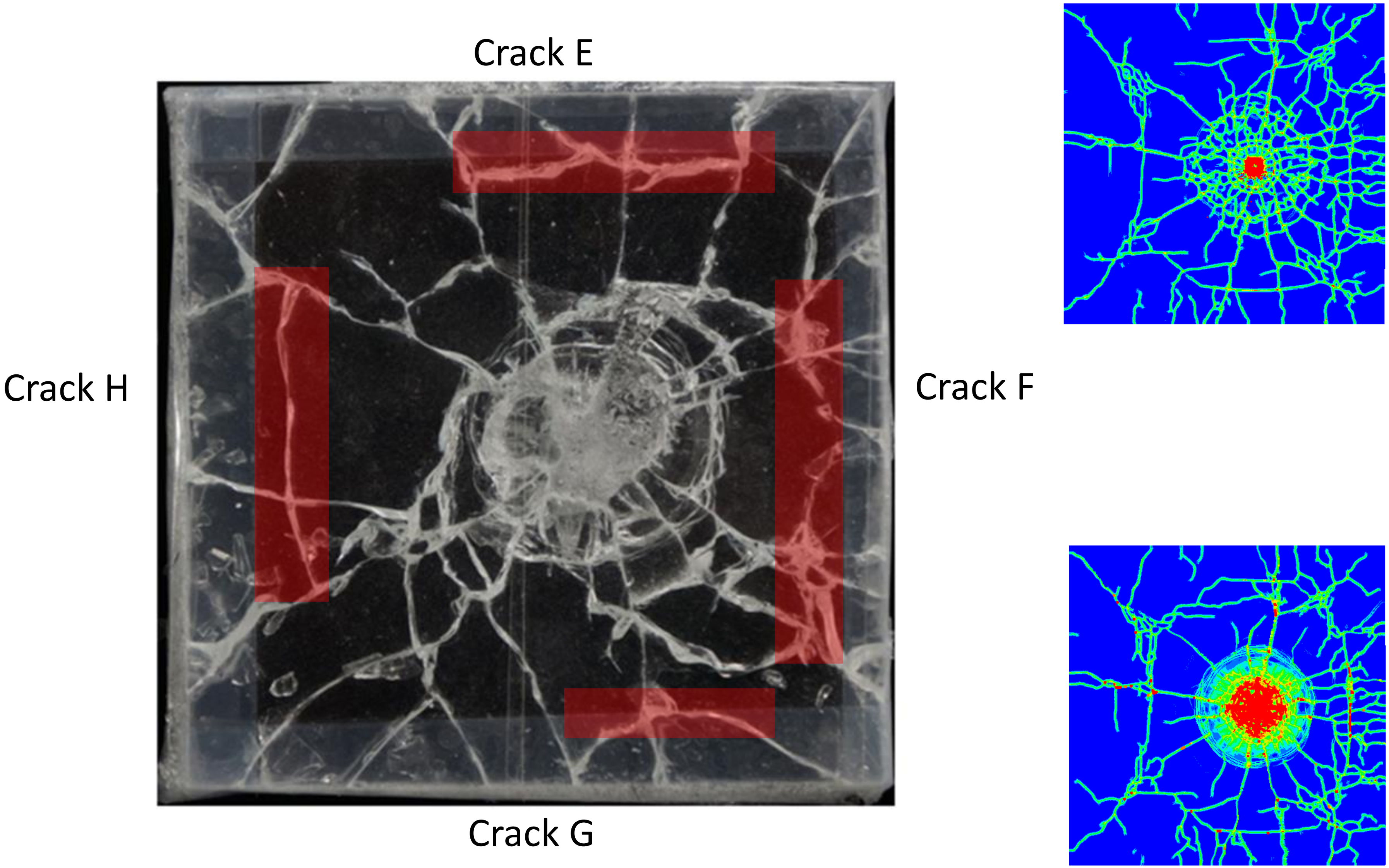
The fragment and the glass chip (view from bottom side)



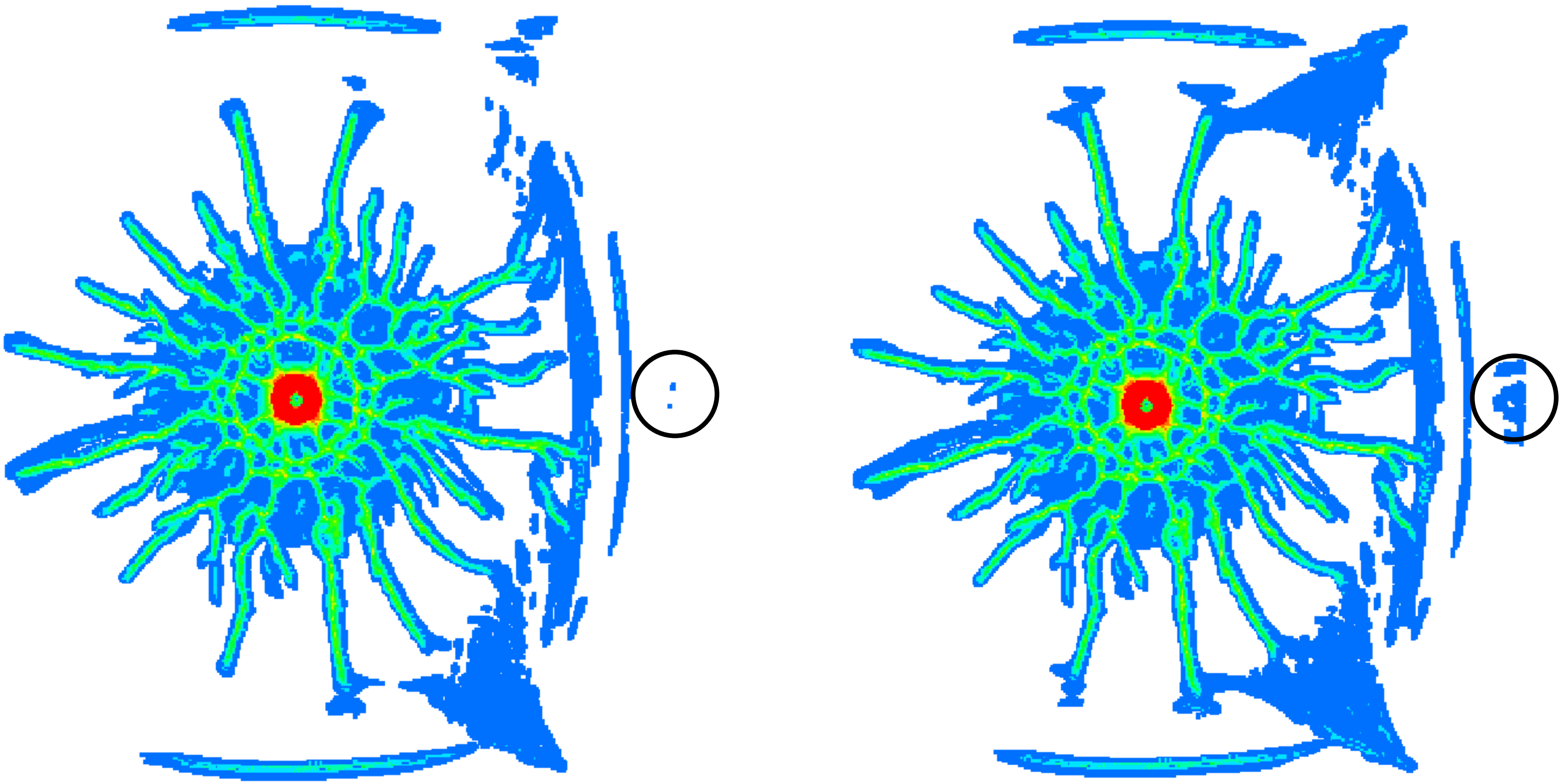
The thinner part of the chip is away from impact center, while the thicker part is closer to impact center.

Out-of-plane nodal velocities

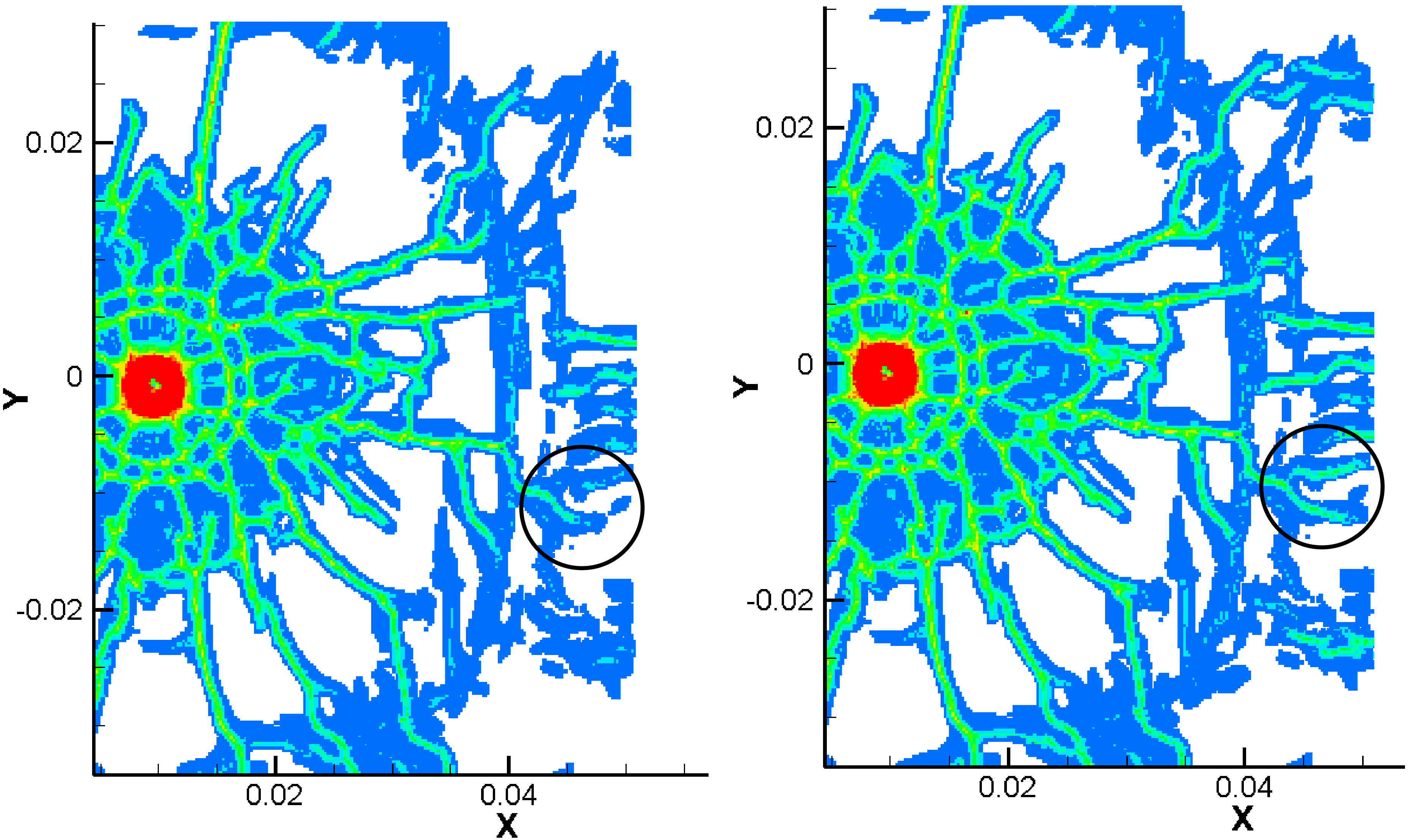


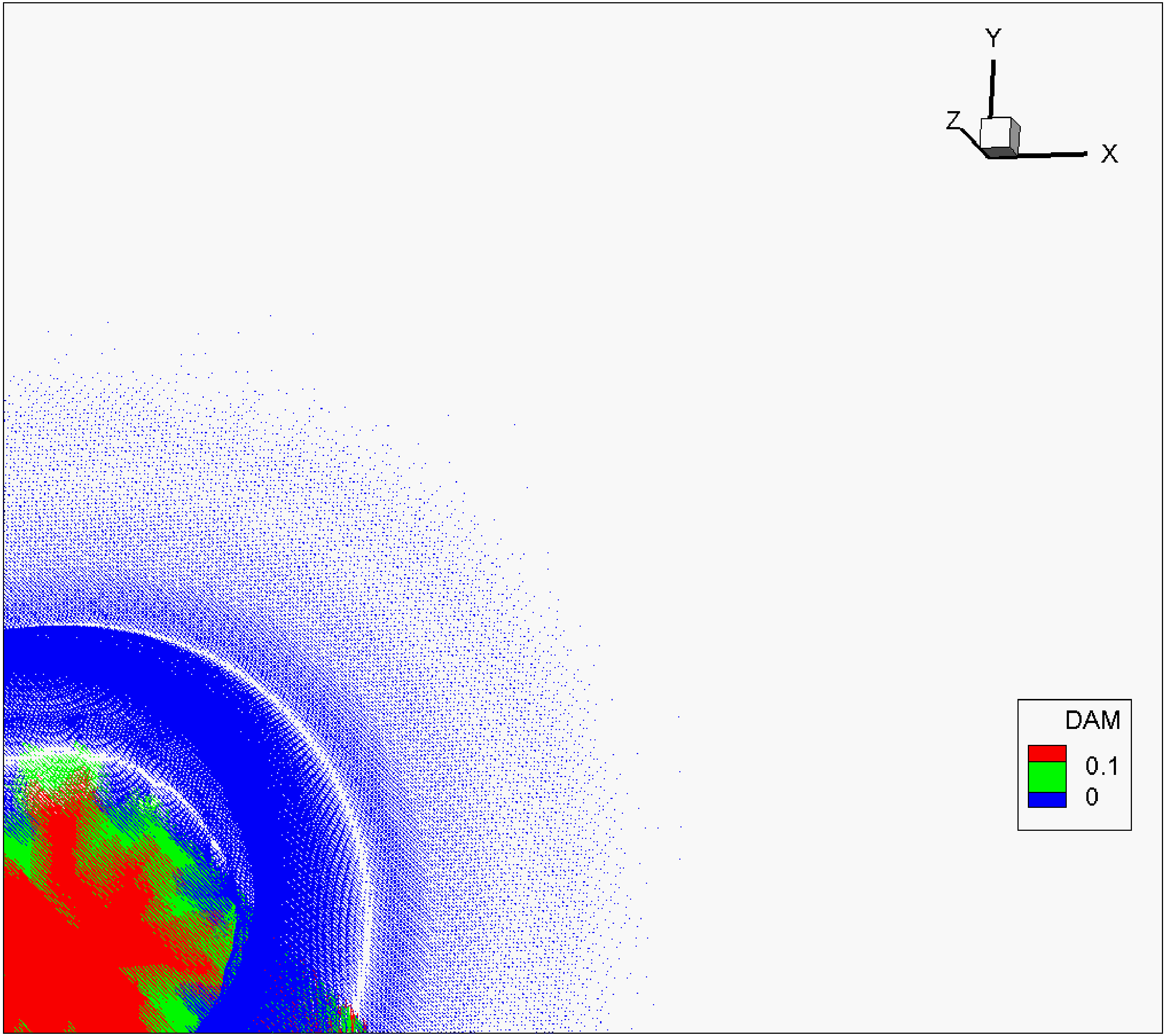

Crack E
Crack H
Crack F
Crack G

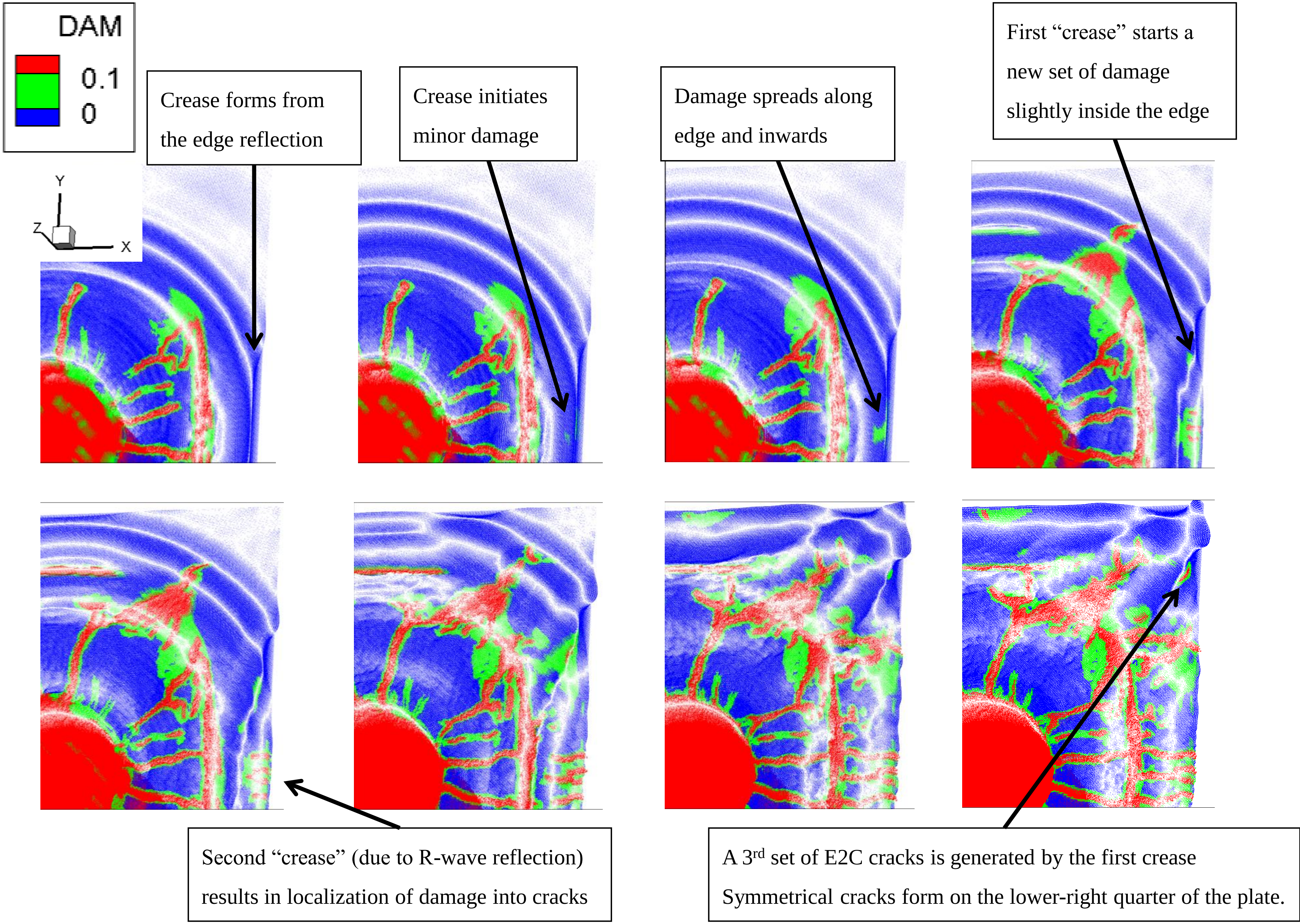
Boundary cracks, Edge-to-Center (E2C) cracks



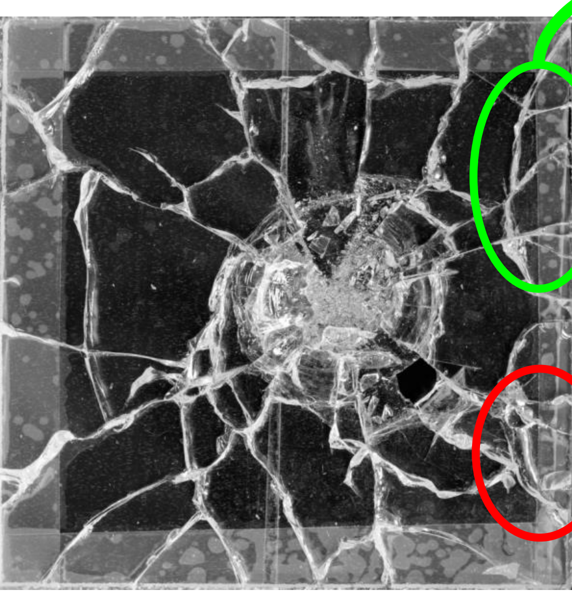
Fake branching

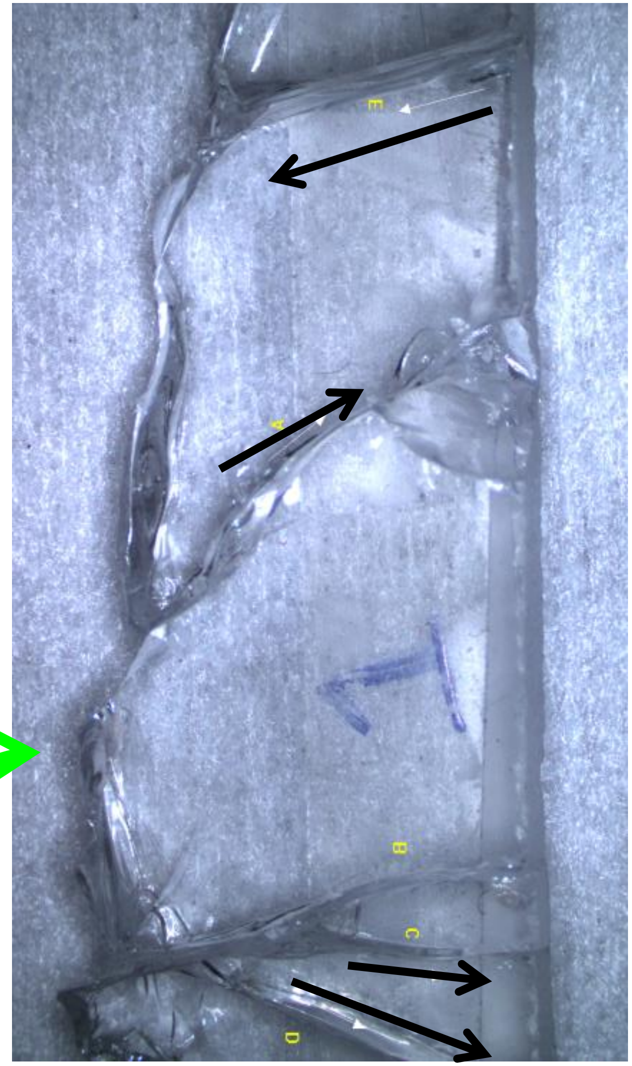


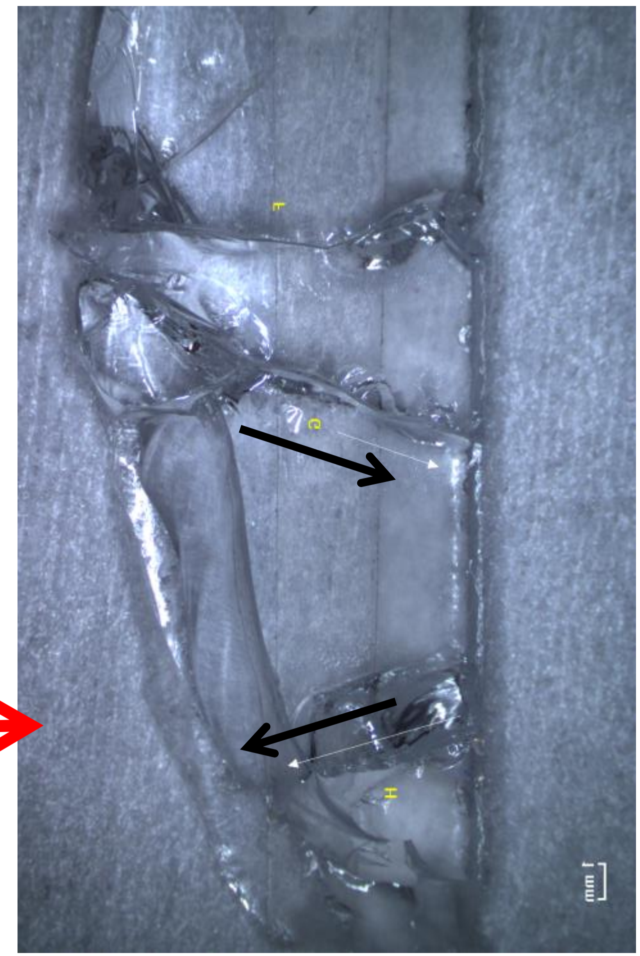

Y
Z
X
DAM
0.1
0

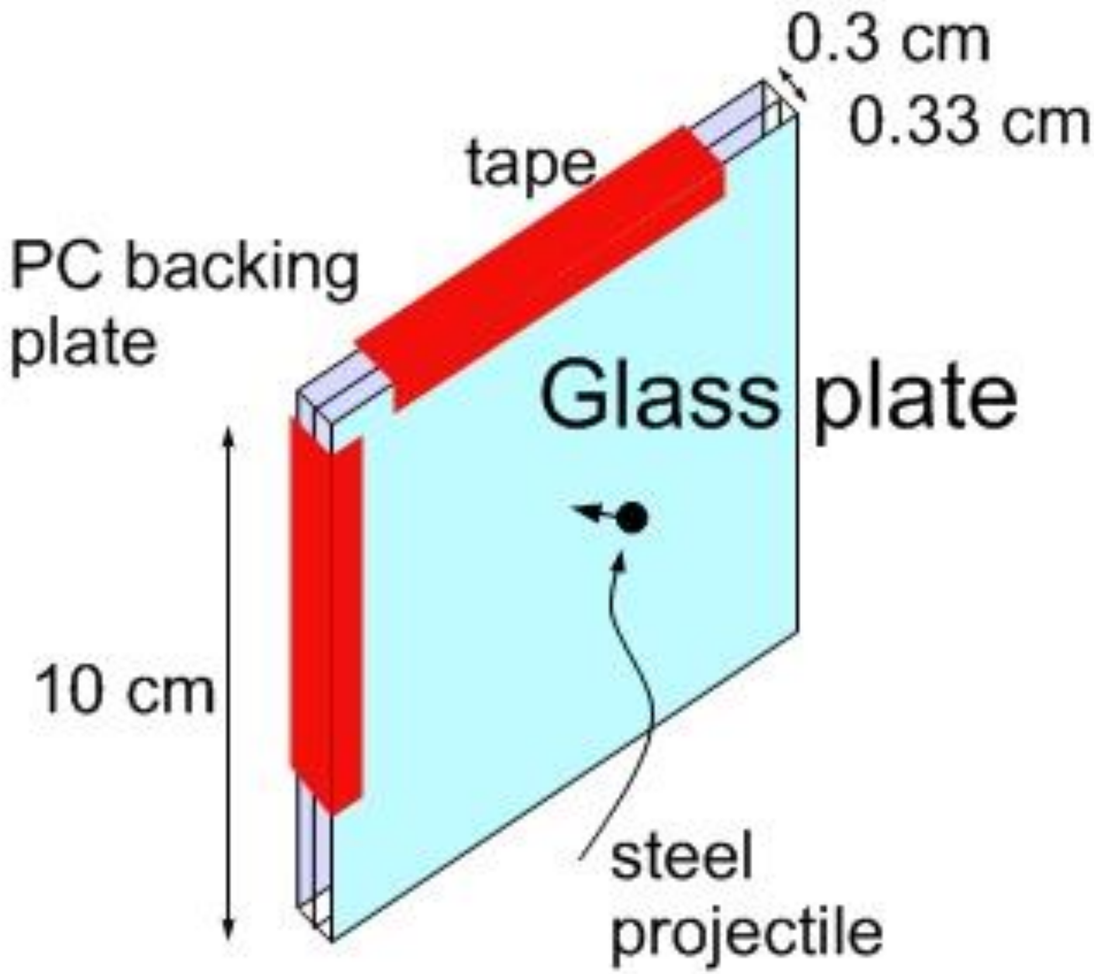

DAM
0.1
0
Y
Z
X
Crease forms from the edge reflection
Crease initiates minor damage
Damage spreads along edge and inwards
First "crease" starts a new set of damage slightly inside the edge
Second "crease" (due to R-wave reflection) results in localization of damage into cracks
A 3rd set of E2C cracks is generated by the first crease
Symmetrical cracks form on the lower-right quarter of the plate.

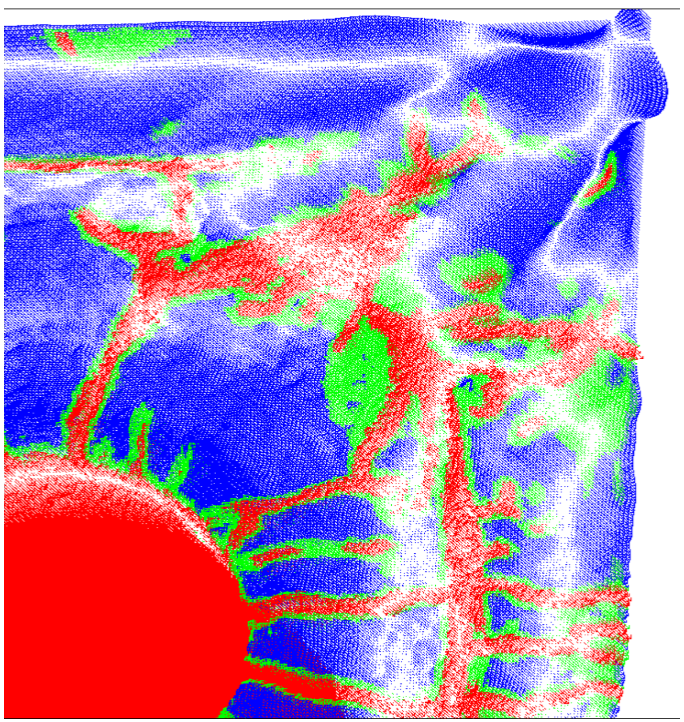
Jared Wright (ARL) fractography results








0.3 cm
0.33 cm
tape
PC backing
plate
Glass plate
10 cm
steel
projectile

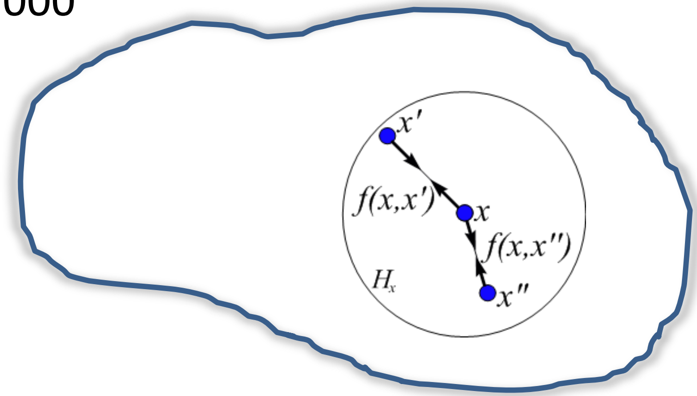


Conclusions

- The simplest peridynamic model (horizon size is 1 mm) with a relatively coarse grid (grid spacing is 0.25 mm) captures fine roughness of a crack surface which forms when Hertzian fracture cone is deflected by stress waves.
- Location of high roughness on surface of crack parallel to strike face in simulations is as in experiments (shows crack speeds are computed correctly).
- Edge-to-center cracks produced by incident waves reinforced by reflected waves. Fractography confirms one set of E2C cracks. The rest may be constrained by the side tape used in experiments.
- Transition from ring cracks to radial cracks (driven by Rayleigh wave front).
- At these strain rates/impact speeds and temperatures, nonlocal continuum model is sufficient, no need for atomistic or quantum. Elasticity and damage/fracture control this process.

The Peridynamics formulation

S.A. Silling *JMPS* 2000



Replace the stress-divergence term in

$$\rho \ddot{\mathbf{u}} = div\ \sigma + \mathbf{b}$$

by an integral of forces

$$\rho \ddot{\mathbf{u}} = \int_{H_x} \mathbf{f}\,(\mathbf{u}(\mathbf{x}', t) - \mathbf{u}(\mathbf{x}, t), \mathbf{x}'-\mathbf{x})\, dV_{\mathbf{x}'} + \mathbf{b}$$

$\mathbf{f}(\eta, \xi)$ is a pairwise force (force density on particle x due to particle x')

$\xi = x' - x$ (original relative position)

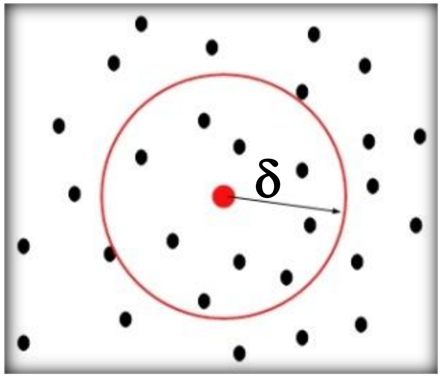
$\eta = (u' + x') - (u + x) = \Delta u + \xi$ (current relative position)

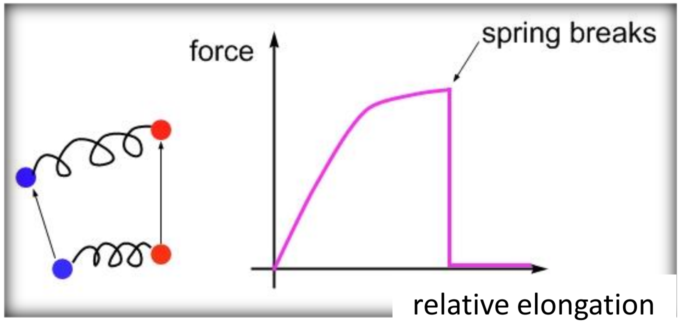
A linear micro-elastic material

$$\mathbf{f}(\eta, \xi) = \frac{\partial w(\eta, \xi)}{\partial \eta}$$

$$w(\eta, \xi) = \frac{c(\|\xi\|)\, s^2\, \|\xi\|}{2}, \qquad s = \frac{\|\xi + \eta\| - \|\xi\|}{\|\xi\|}$$

relative elongation





Introduce **damage**

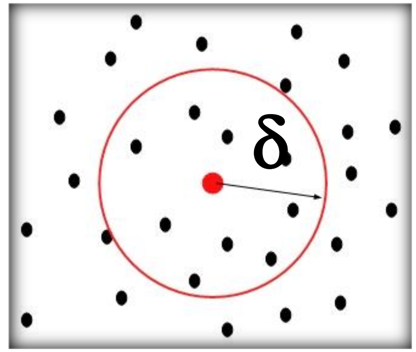
$$\hat{f}(s, |\xi|, x, t) = \bar{f}(s, |\xi|)\ \mu(\xi, x, t)$$

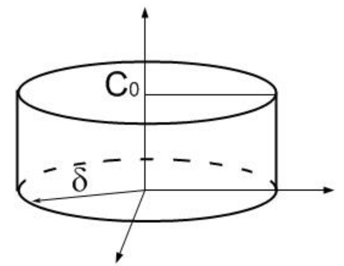
with $\mu(\xi, x, t)$ a history - dependent scalar 0 - 1 function

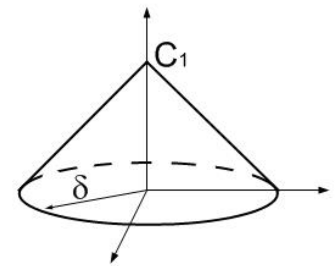
$$\mu(\xi, x, t) = \begin{cases} 1 & \text{if } s < s_0 \text{ for all } 0 \le \tau \le t \\ 0, & \text{otherwise} \end{cases}$$

(**Silling and Bobaru** *Int. J. Nonlin. Mech.* **2005**)

Link to measurable Material Properties







<u>Micromodulus function:</u>

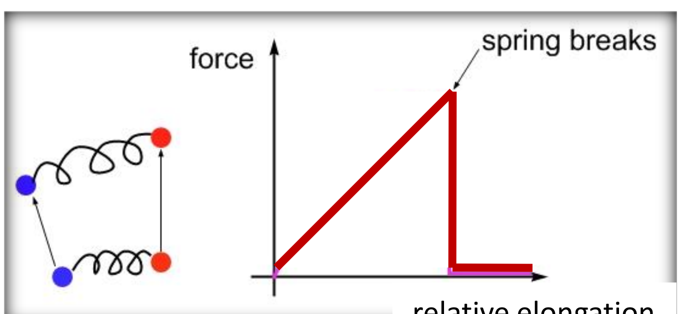
match the strain energy
for a homogeneous
deformation to classical elasticity

$$c(\|\underline{\xi}\|) = \begin{cases} \dfrac{24E}{\pi\delta^3 t(1-2v)(1+v)}\left(1-\dfrac{\|\underline{\xi}\|}{\delta}\right), & if \quad \|\underline{\xi}\| \le \delta \\ 0, \quad if \quad \|\underline{\xi}\| > \delta \end{cases}$$

Bobaru et al (2009) IJNME

Convergence to classical , local solutions

<u>Critical relative elongation</u>: match the energy/m^2 to separate into two halves



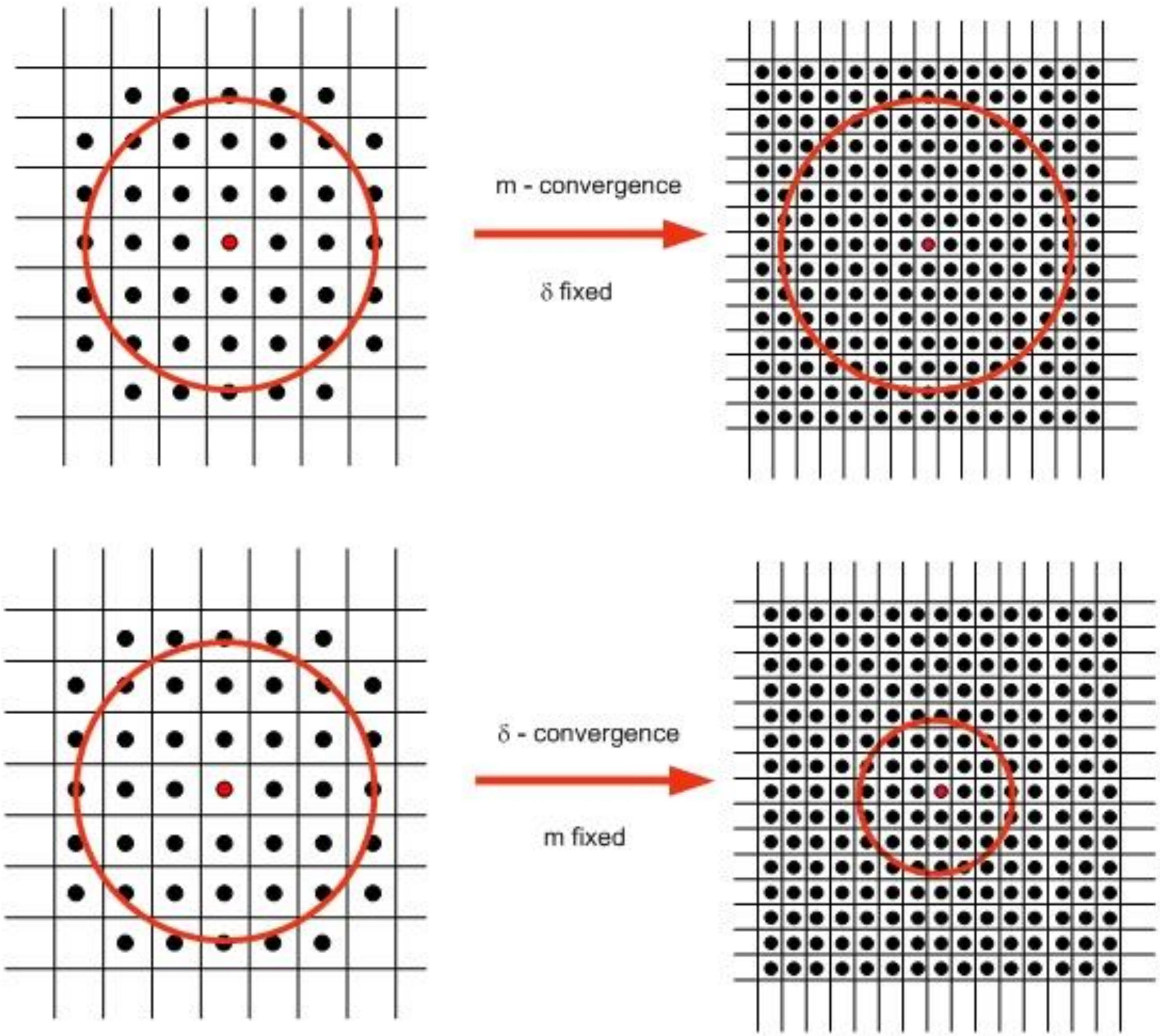
$$s_0 = \sqrt{\frac{5\pi G}{18\kappa\delta}}$$

$$s_0 = \frac{1}{3}\sqrt{\frac{5G}{\kappa\delta}}$$

Silling & Askari, *Comp. and Struct.* 2005 (3D)

E. Emmrich, 2006 (2D)

Convergence in Peridynamics



$$m = \frac{\delta}{\Delta x}$$

Material parameters

Soda lime glass;

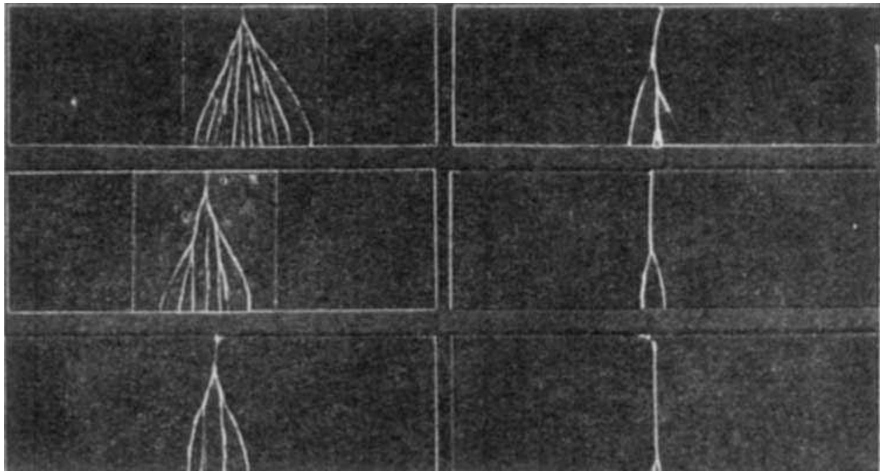
- Bulk modulus: 43 GPa
- elastic modulus: 72GPa
- density: 2440kg/m^3
- Poisson's Ratio: 0.22-0.24 (0.25*)
- longitudinal wave speed: 5432m/s
- shear wave speed: 3477m/s
- Rayleigh wave speed: 3139m/s

Bond-based EMU model matches the bulk modulus 43 GPa

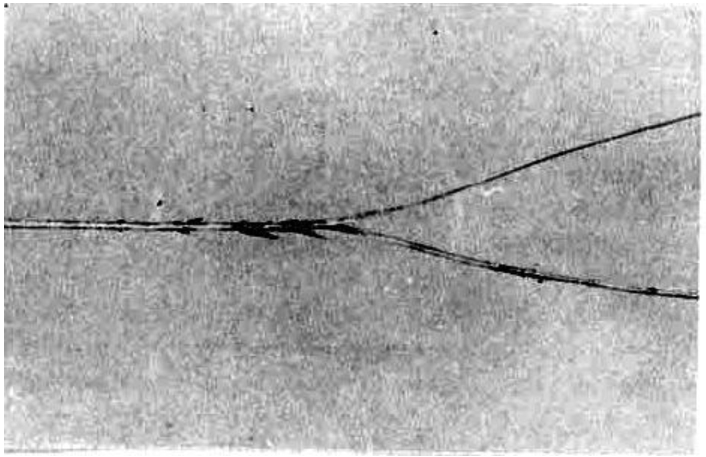
- Longitudinal wave: 5632m/s
- shear wave: 3252m/s
- Rayleigh wave: 2990m/s

Dynamic fracture/Crack branching

- Experimental observations:
  - Branching occurs when a critical stress intensity factor is reached not when a maximum velocity is reached.
  - The crack is at or near its maximum velocity for a considerable path length before branching.
  - Roughening of the crack surface before branching reported in all experiments
  - Wave propagation can influence crack branching, curving.



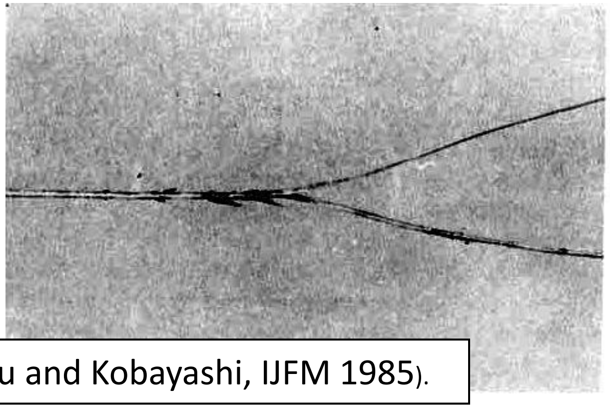
Bowden et al, Nature 1967.



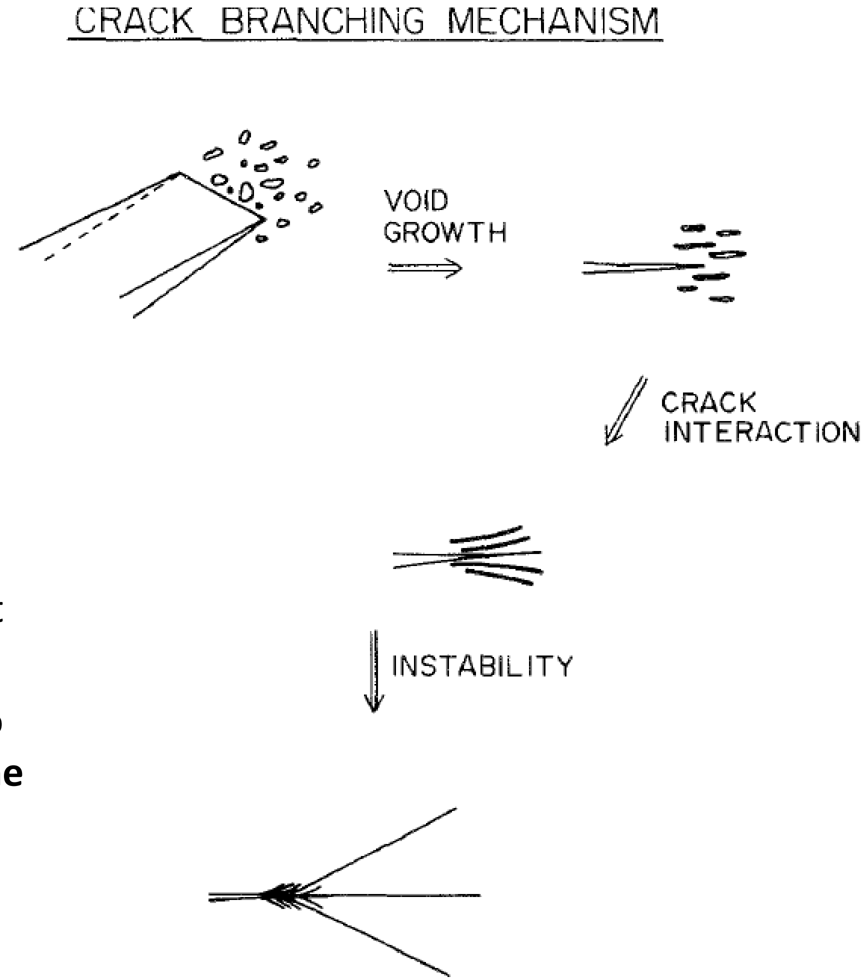
Crack branching in a single edge notch specimen of homalite-100 (**Ramulu and Kobayashi, IJFM 1985**).

Possible mechanisms in Crack Branching

- Ravi-Chandar and Knauss, IJF (1984): microvoids/microcracks (glassy polymers)
- Hull (1994): mixed-mode loading near the crack tip (formation of lances, roughness)



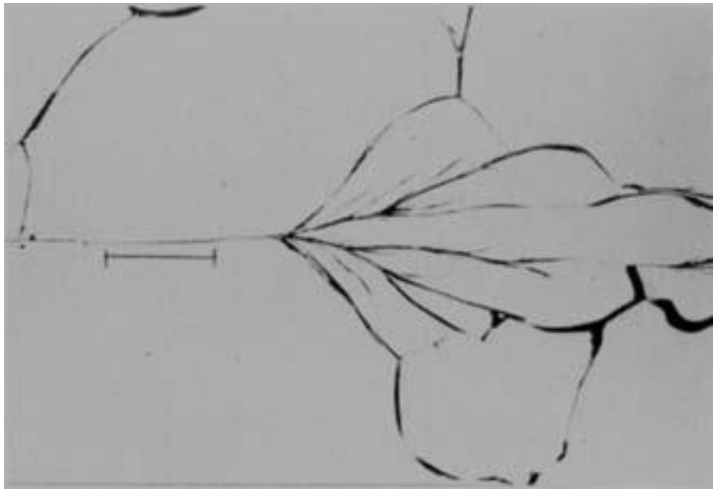
(Ramulu and Kobayashi, IJFM 1985).

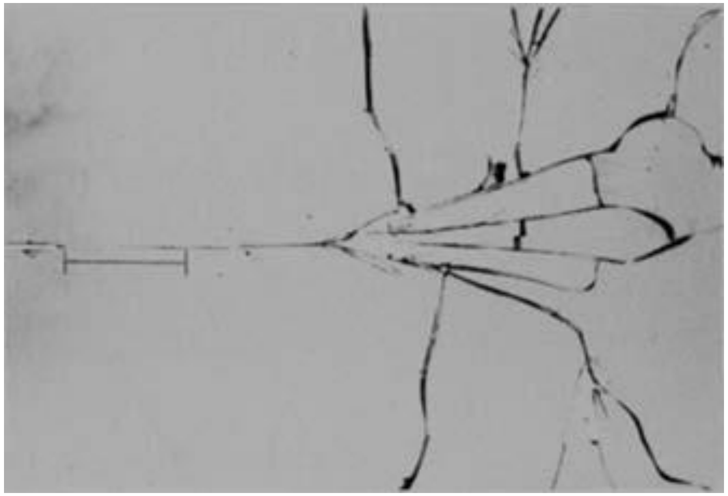


- Roughening of the crack surface before branching.
- Branching occurs when a critical SIF is reached, not when a maximum velocity is reached.
- When a crack reaches a critical SIF it splits into two or more branches, each propagating **with about the same speed as the parent crack, but with a much reduced process zone** (Ravi-Chandar, 2004).

Influence of stress waves

Ravi-Chandar and Knauss, IJF (1984)

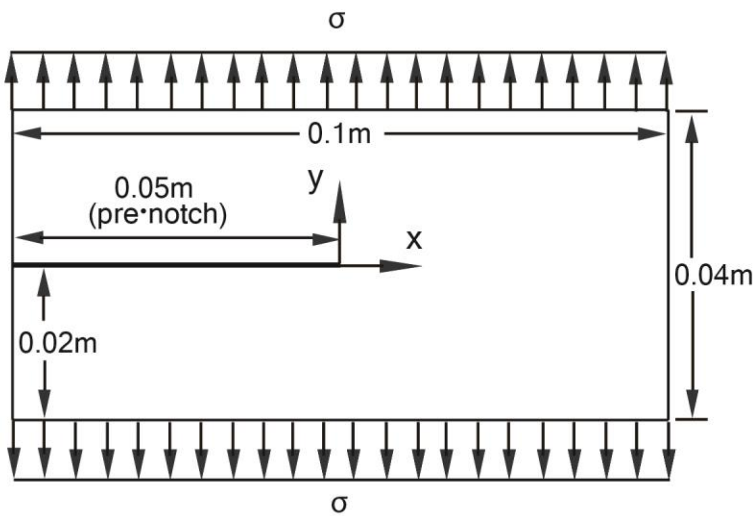


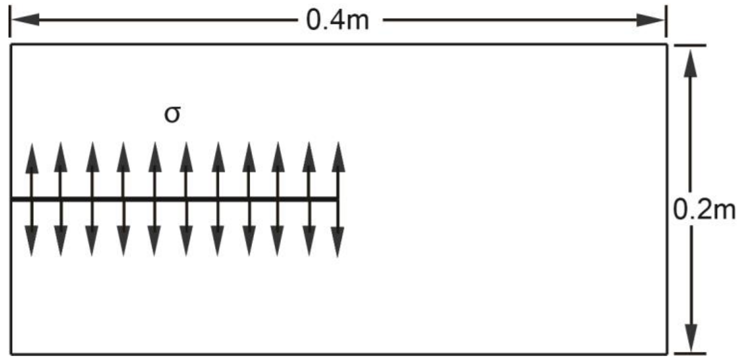


- Stress waves:
  - influence the shape and propagation speed of dynamic cracks.
  - can generate secondary cracks perpendicular to the original cracks
  - can induce crack branching (but are not necessary for branching)

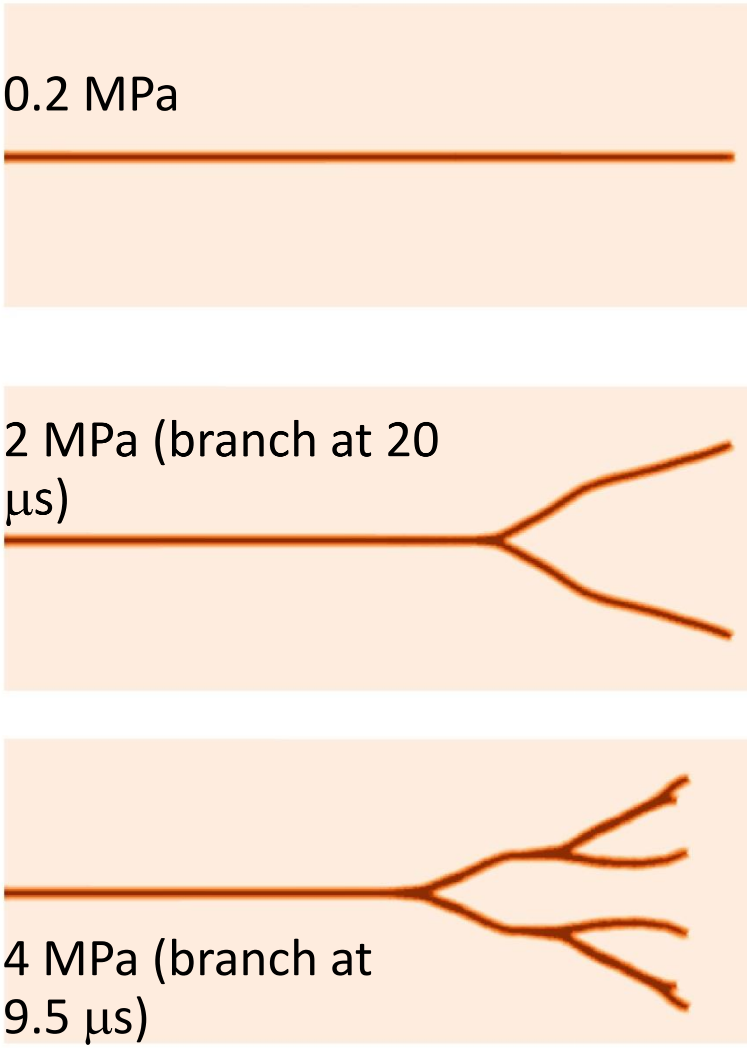
Understand crack branching with peridynamics

- Role of loading conditions (waves).
- Effect on branching angle, crack propagation speed, etc.
- What happens near the crack tip?

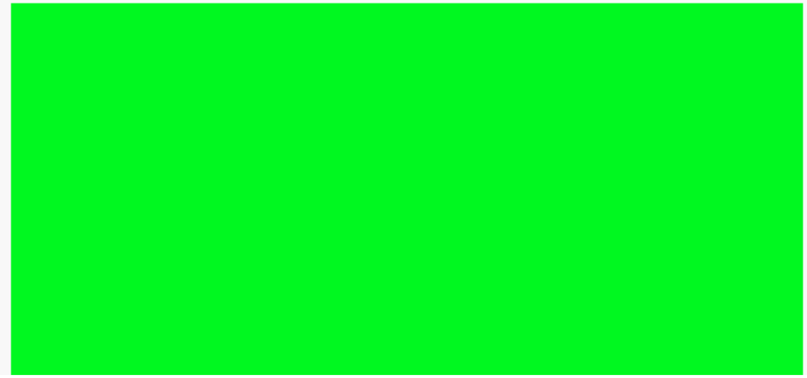


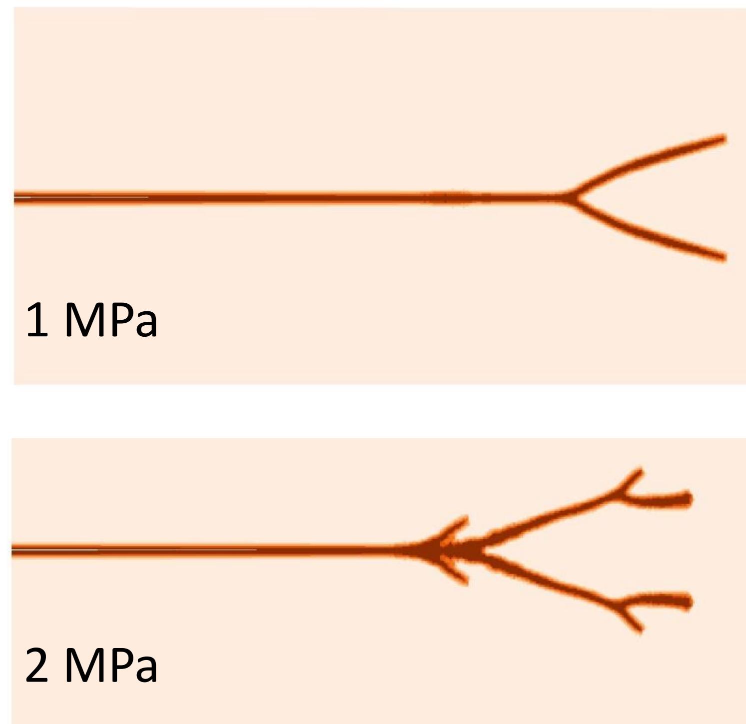


Sudden loading on boundaries



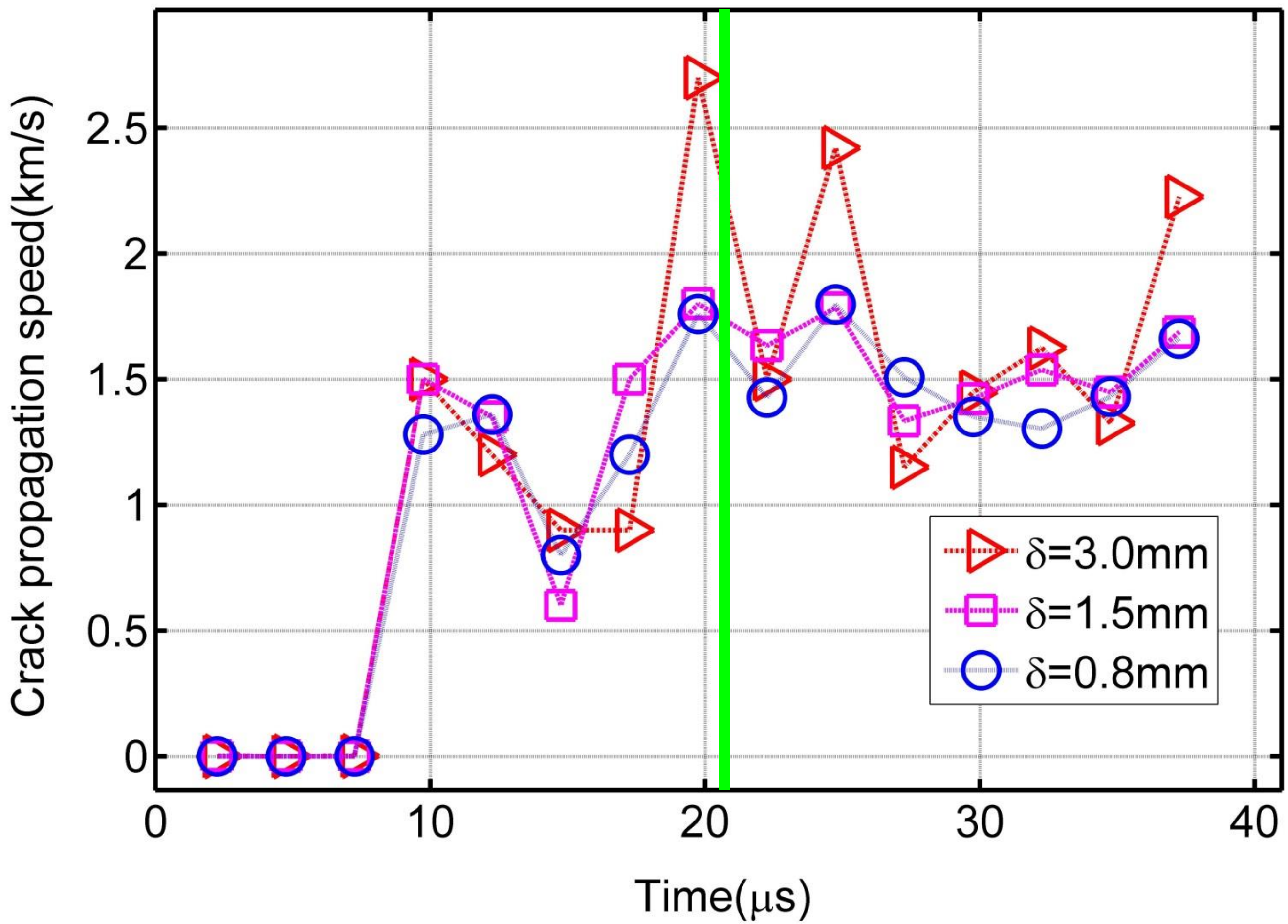
glass





homalite

How to choose the horizon size?

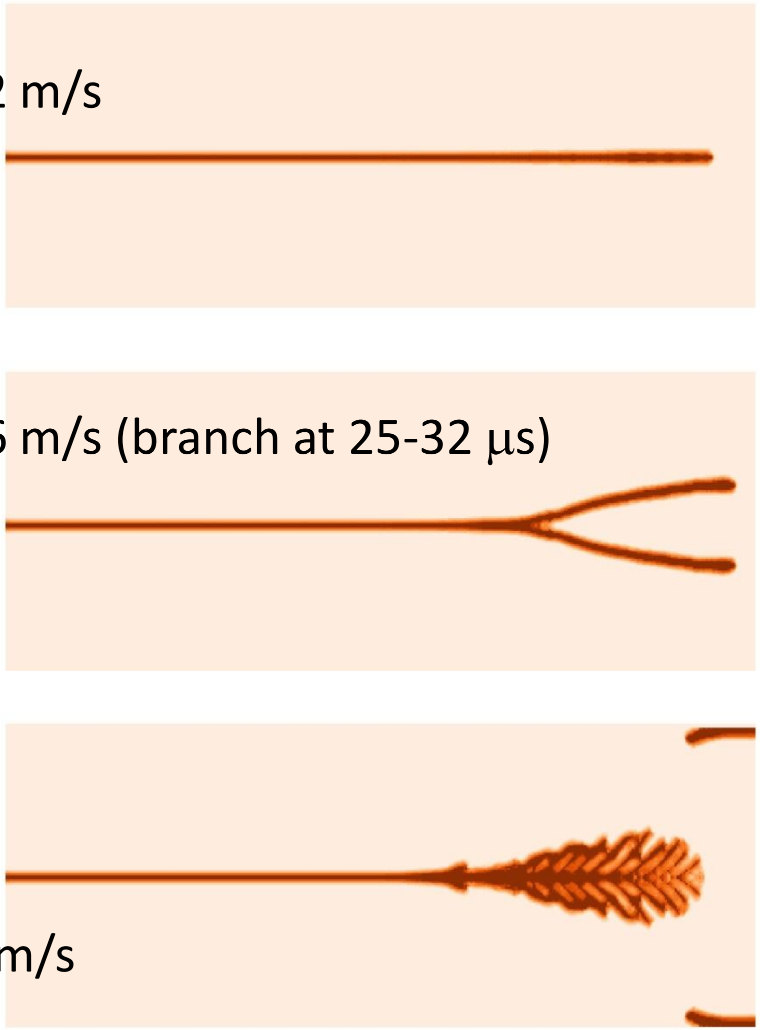


Constant speed separation of boundaries

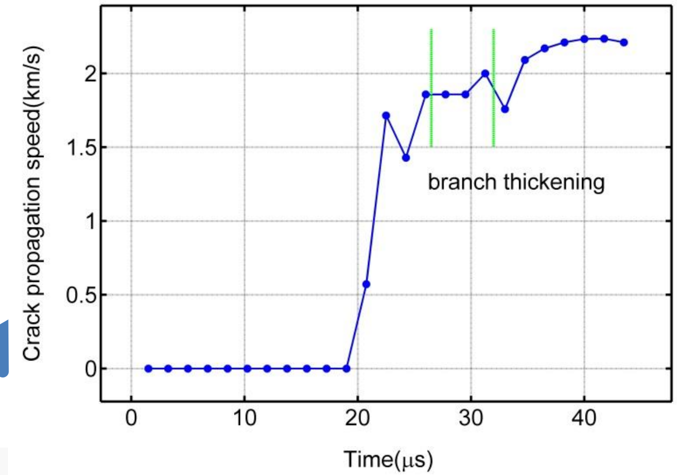
0.02 m/s

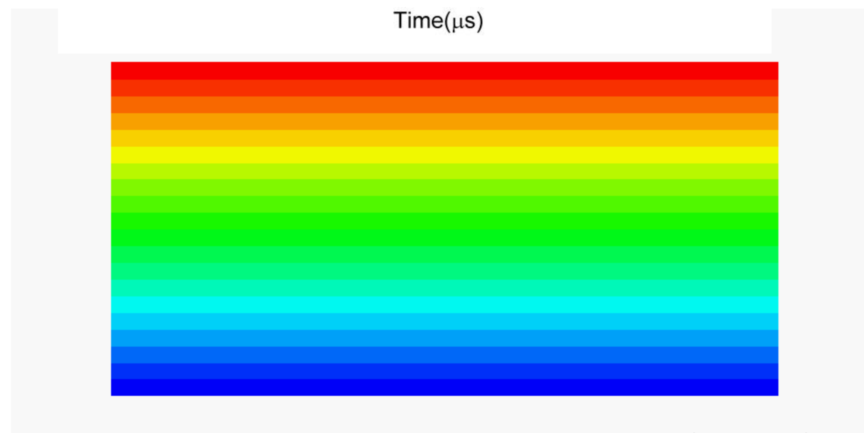
0.06 m/s (branch at 25-32 μs)

0.2 m/s

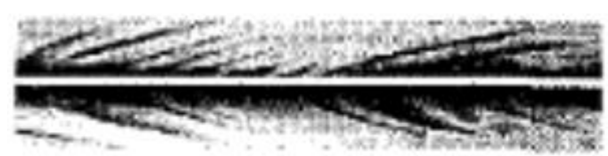


glass





homalite



PMMA (Fineberg 2006)

Comparisons with experiments

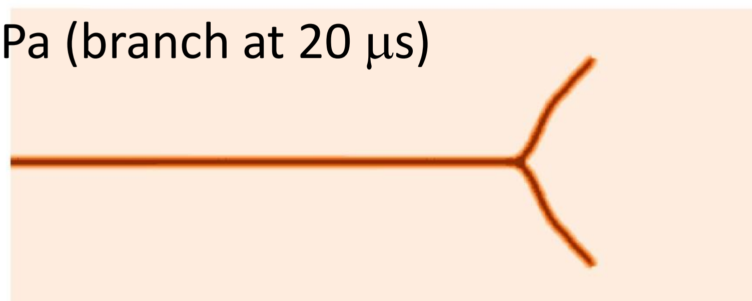
Ratio of **speed at branching** to the Rayleigh wave speed.

Loading	Soda-lime glass		Homalite	
	Loading Magnitude	V/Cr	Loading Magnitude	V/Cr
Stresses on BC	2MPa	0.52	1MPa	0.46
	4MPa	0.68	2MPa	0.46
Stress on crack surface	3MPa	0.42	2MPa	0.25
	6Mpa	0.44	4MPa	0.55
Velocity BC	0.06m/s	0.58	0.2m/s	0.59
	0.2m/s	“fish bones”	1m/s	“fish bones”

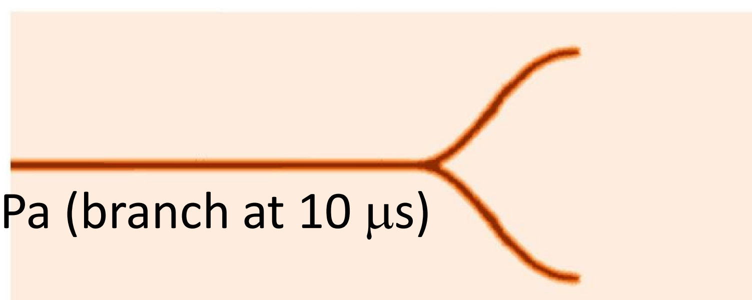
Avoid wave interactions with crack surface loading

0.5 MPa

3 MPa (branch at 20 μs)



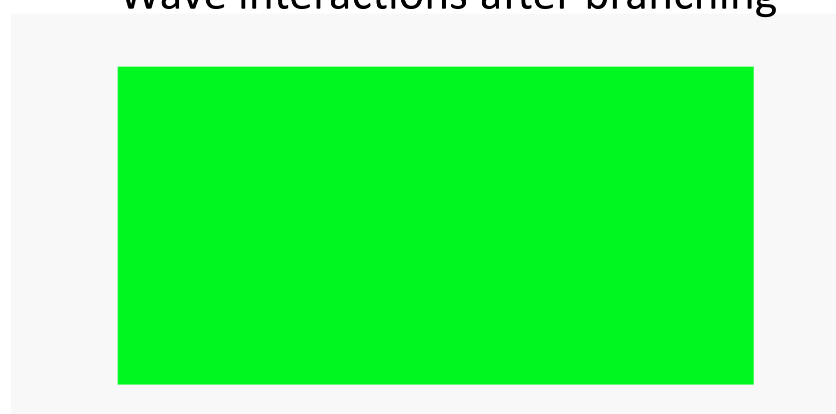
6 MPa (branch at 10 μs)



glass

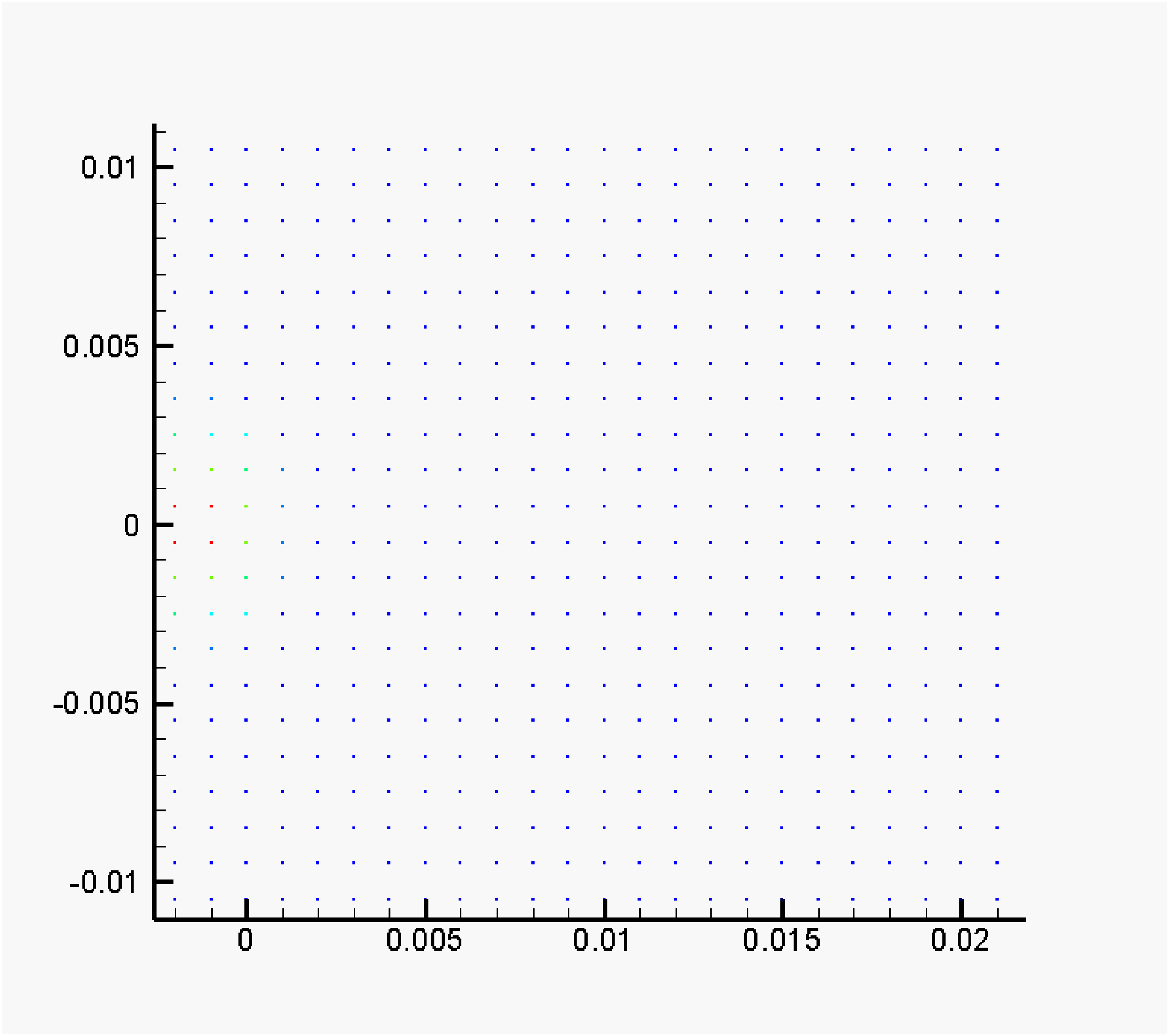
Wave interactions before branching

Wave interactions after branching

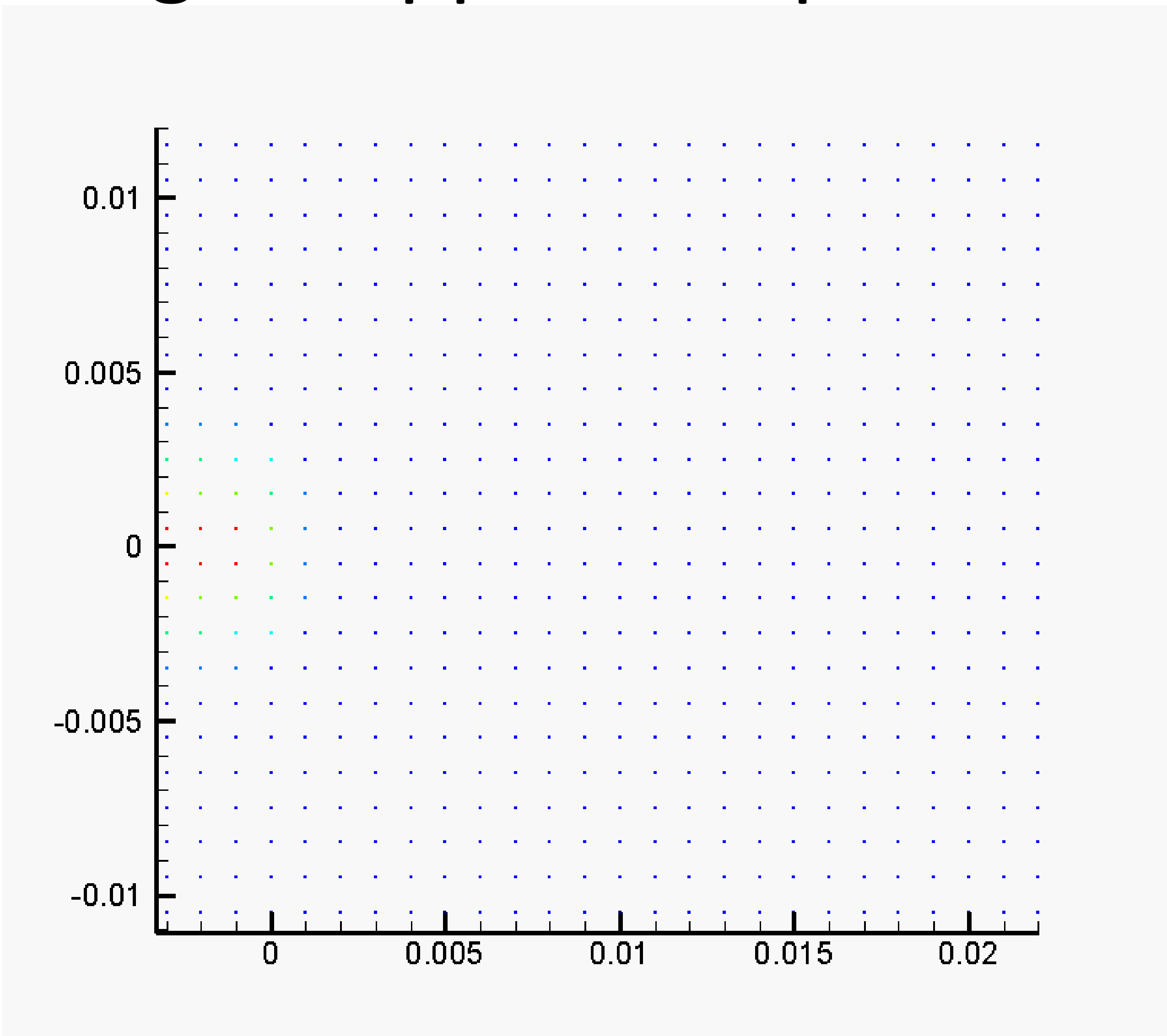


homalite

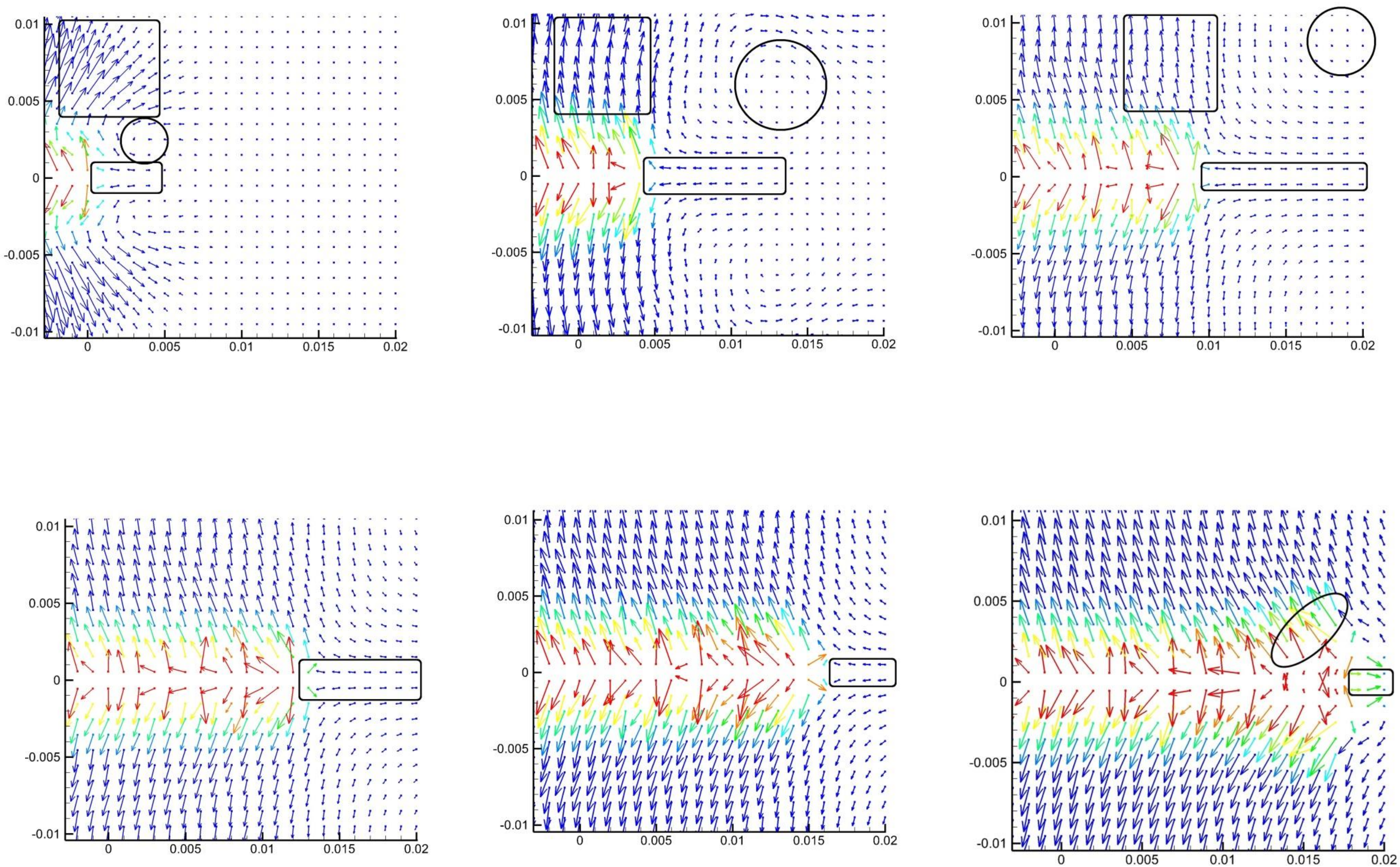
Why/How do cracks branch? (in this model)



At higher applied amplitude loading



Migration of damage away from the crack line

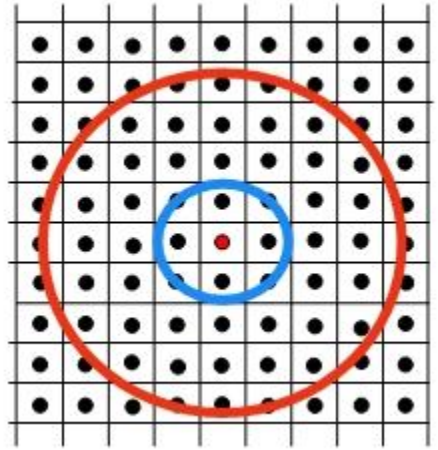


Why do local models have trouble with brittle failure?

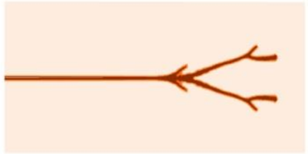
- Need to model the roughness before branching to dissipate the "right" amount of energy before branching.
- In dynamic fracture, everything that follows a "wrong" move is wrong.
- Models that use crack surface tracking (one surface splits in two) need extra conditions to mimic the actual dissipation in the process zone. What conditions?
- Nonlocality helps in allowing damage to "grow" in a preferential direction as a new fracture surface with a much reduced process zone.

A “local nonlocal” PD model

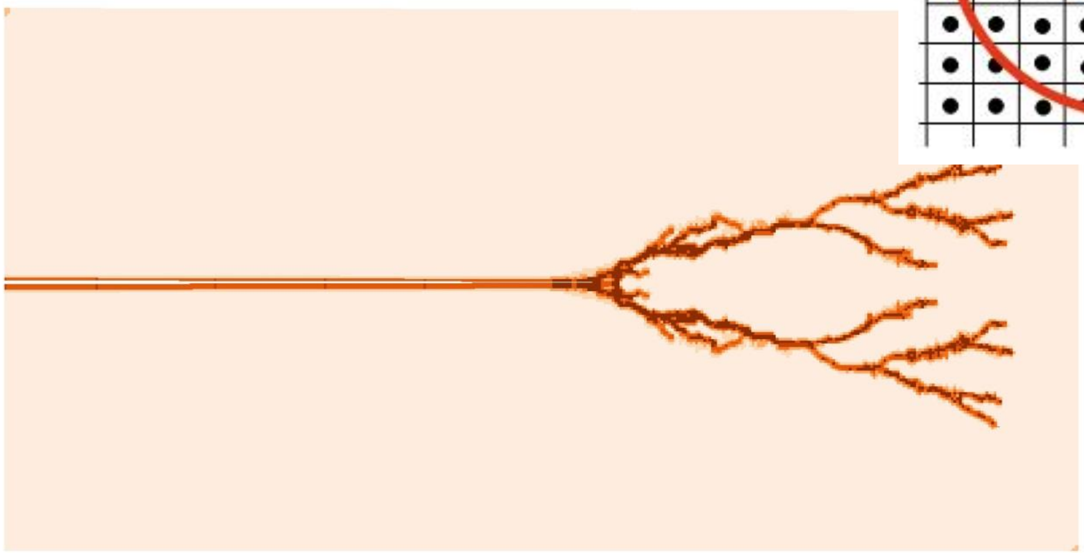
$$m = \frac{\delta}{\Delta x}$$

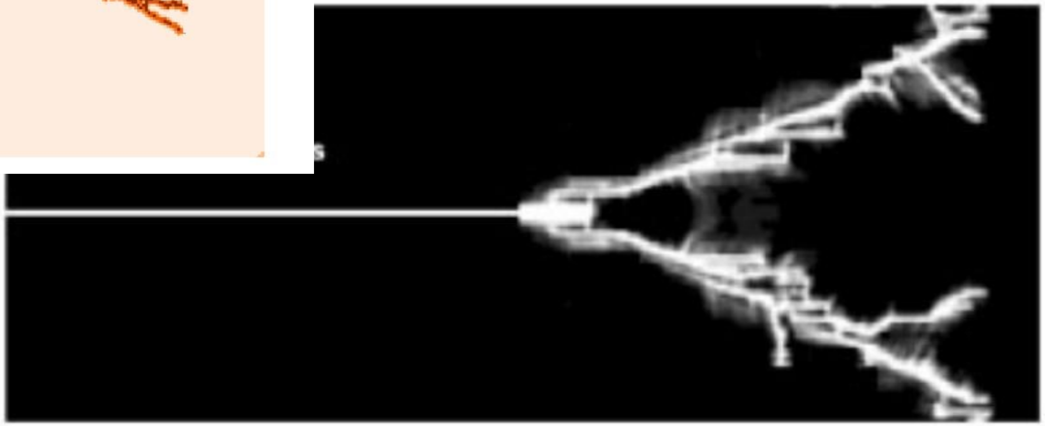


m=4.01



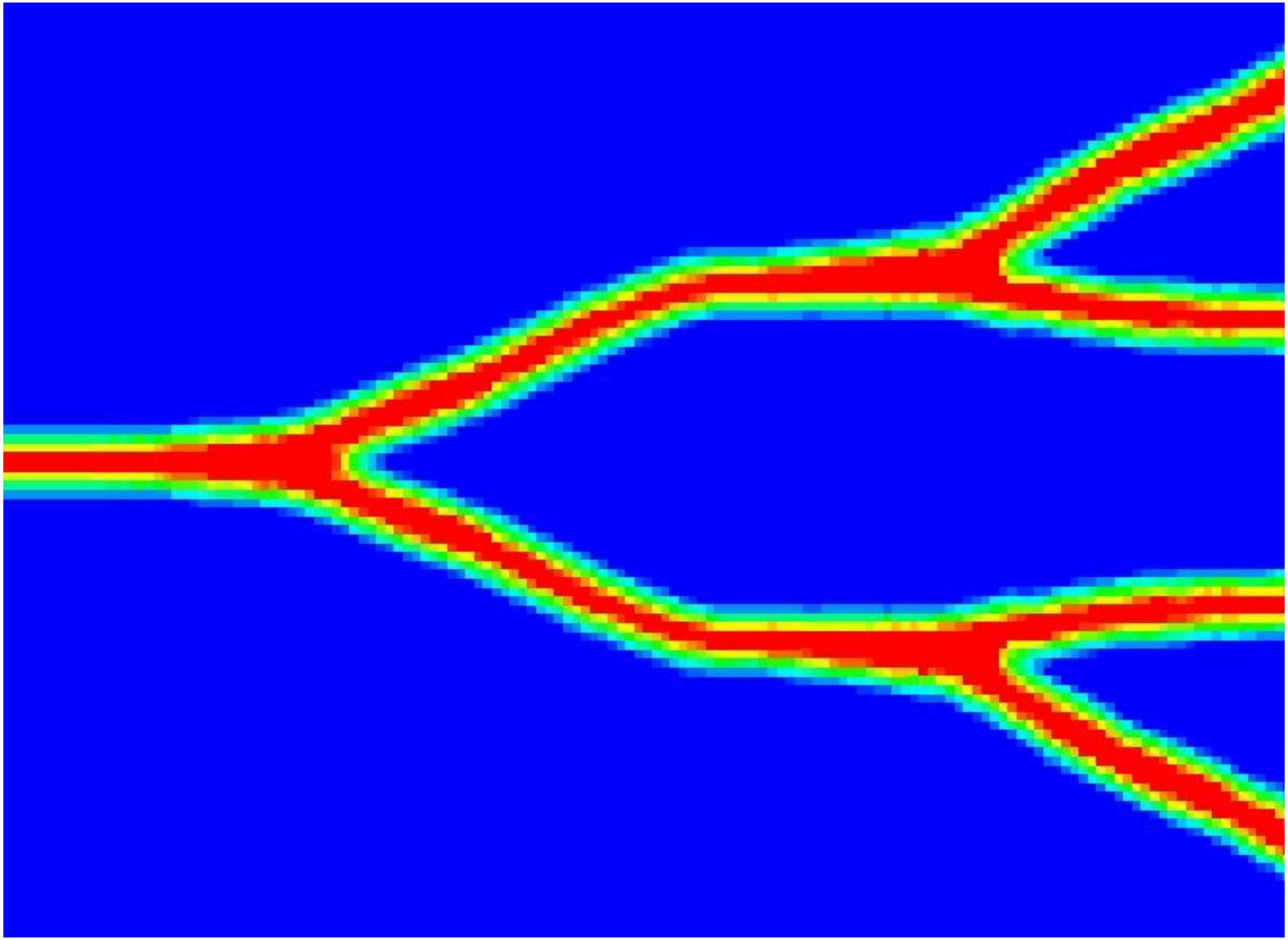
m=1.45





Rabczuk and Belytschko, IJNME 2004)

Thickening of the process zone before branching



Nonlocality and length-scales

- Peridynamic nonlocal damage is effective in modeling the process zone evolution. Once nonlocal scale (horizon) is on the scale of the process zone, predictive results.
- Migration of damage away from the crack line caused by waves; leads to mixed-mode loading conditions and **branching of the process zone.**
- Dynamics of strain energy delivery into the process zone controls crack branching.
- Local models need to somehow insert this in the formulation. Difficult to postulate a criterion, since it depends on local stress/strain conditions (waves, geometry).